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IMPACT OF VARIABLE RENEWABLE PRODUCTION ON ELECTRICITY PRICES IN GERMANY: A MARKOV SWITCHING MODEL

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Cyril Martin de Lagarde[1] Frédéric Lantz[2]

Abstract

This paper aims at assessing the impact of renewable energy sources (RES) production on electricity spot prices. To do so, we use a two-regime Markov Switching (MS) model, that enables to disentangle the so-called "merit-order effect" due to wind and solar photovoltaic productions (used in relative share of the electricity demand), depending on the price being high or low. We find that there are effectively two distinct price regimes that are put to light thanks to an inverse hyperbolic sine transformation that allows to treat negative prices. We also show that these two regimes coincide quite well with two regimes for the electricity demand (load). Indeed, when demand is low, prices are low and the merit-order effect is lower than when prices are high, which is consistent with the fact that the inverse supply curve is convex (i.e. has increasing slope). To illustrate this, we computed the mean marginal effects of RES production and load. On average, an increase of 1GW of wind will decrease the price in regime 1 (resp. 2) by 0.77€/MWh (resp. 1€/MWh). The influence of solar is slightly weaker, as an extra gigawatt lowers the price of 0.73€/MWh in period 1, and 0.96€/MWh in regime 2. On the contrary, if the demand increases by 1GW in regime 1 (resp. 2), the price increases on average by 0.93€/MWh (resp. 1.18€/MWh). Although we made sure these marginal effects are significantly different from one another, they are much more variable than the estimated coefficients of the model. Also, note that these marginal effects are only valid inside each regime when there is no switching. The latter regime partly corresponds to the high load regime, at the exception of periods during which RES production is high. The impact on volatility could also be observed: the variance of the (transformed) price is higher during the high-price regime than in the low-price one. In addition to the switching of the coefficients, we allowed the probabilities of transition between the two regimes to vary over time, following a binomial logistic link with the relative share of RES production. This analysis shows that both wind and solar productions have a significant impact on the switching mechanism, especially on the probability of switching from the high-price regime to the low-price one, and consequently on the expected duration of each regime. However, the effect of wind production on the probabilities is much higher than the effect of solar production, whereas they have a rather similar marginal effect on the price. Finally, although the regimes are sometimes highly correlated with some hours of the day, their endogenous determination (opposed to a semi-deterministic approach with dummy variables, for example) gives flexibility and keeps the model parsimonious.

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1. INTRODUCTION

1.1. Context

The development of renewable energy sources (RES) is often justified by the need to address global warming, through the reduction of green-house gases emissions, and is also led by the will to reach energy independence in fossil and fissile fuel-dependent countries. In the electricity field, main RES are wind power and solar photovoltaic (PV). These technologies are spreading throughout Europe, which has announced RES targets for the next decades: 20% in the final energy consumption by 2020 (legally binding) and 27% by 2030 (not legally binding). To reach these goals, RES need to be subsidised, since they would not be competitive on electricity markets otherwise. The subsidisation of these energies aims at internalising the learning effect, i.e. the decrease of their cost along with their development. This is a positive externality that is by definition not taken into account by the market, and which would lead to too few investments in such technologies if not taken care of.

However, the development of electric RES challenges the current design of electricity markets. Indeed, it was originally meant to reflect the short-term production cost of electricity via the system marginal price, i.e. the marginal cost of the last unit needed to meet the demand. While marginal costs were traditionally driven by fuel costs, such as coal, gas, oil, or uranium, wind and photovoltaic have on the contrary (almost) no marginal cost. Therefore, they tend to lower the spot price when they are producing, which is commonly known as the “merit-order effect”. In addition, wind and solar energies are intermittent (or variable), albeit with seasonal patterns, while electricity prices are highly seasonal, with seasonality being driven mainly by demand, at the daily, weekly and yearly levels. Hence, RES production is likely to have a different impact on electricity prices, depending on where the supply-demand equilibrium stands. In particular, if the inverse supply curve[3] is locally steep, the impact is expected to be higher than when it is locally flat. In general, the steepness of the inverse supply curve increases with load (i.e. it is convex), as suggested by figure 1, which illustrates the variation of the merit-order effect with the load level. Also, the volatility incurred for these high levels of price is likely to be higher than for low prices, all other things held constant.

These impacts are very important for both investment decisions and energy policies, since they affect the profitability of all power plants, including RES themselves in the long run. Indeed, the current modification of support schemes (e.g. switching from feed-in tariffs to feed-in premiums), and one day their end, will make them face more and more with the market, on which they will need to be profitable as well as other production units. This question is hence highly impacting for the design of RES support mechanisms, but also for the future designs of electricity markets in general, with another big issue being the security of supply, that requires attractiveness for investors. This issue is also relevant for the distribution system operators (DSOs), since the majority of new RES capacity is connected directly to the distribution network. Consequently, the latter needs to be developed and reinforced, and these investments directly depend on the decisions of the producers.

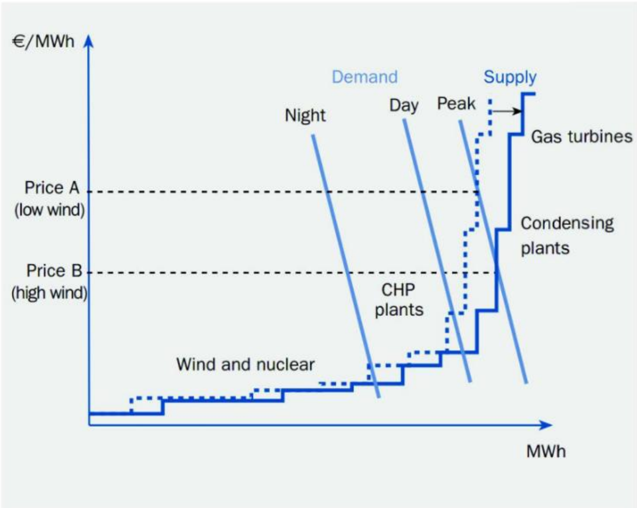
1.2. Aim of the study and literature review

In this paper, we model the impact of variable RES production on electricity prices in Germany, through an econometric analysis of time series, using a Markov (regime-) switching (MS) model. Hence, we first present some literature on the econometric modelling of electricity prices with RES production, before citing some contributions MS models, and some applications to electricity prices.

On the one hand, several authors have already studied the impact of RES production on electricity prices using time-series econometrics. Most of them used ARMA-GARCH models, such as Ketterer (2014) for Germany or Karanfil and Li (Forthcoming, 2017) for Denmark, just to cite a few recent ones. All these analyses found a significant impact of RES generation on electricity prices and

[3] The inverse supply function S^{-1} gives the price p in function of the supply quantity q. The inverse supply curve is thus $p = S^{-1}(q)$.

volatility, but this impact is necessarily averaged over the whole time series. Some other methods have also been used, such as Paraschiv, Erni, and Pietsch (2014), who estimated the impact of RES production on electricity prices in Germany for each hour of the day, using a dynamic fundamental state-space model in which the coefficients follow a random walk and react to new information; or Woo et al. (2011), who used a partial-adjustment linear regression model with binary dummies for hour, month and weekday, to study the impact on the electricity prices in Texas.



Source: Risø DTU

Figure 1. Differentiated merit-order effect

On the other hand, the literature on the modelling of electricity prices through econometric approaches is very wide, but our review will focus on MS models only. These were first introduced by Goldfeld and Quandt (1973), and popularised by J. D. Hamilton (1989, 1990) in particular. Markov regime-switching models were further developed by J. Cai (1994), J. Hamilton and Susmel (1994) and Gray (1996), who introduced ARCH and GARCH switching models. Let us finally cite Krolzig (1997), who developed vector autoregressive MS models (MS-VAR).

Since electricity cannot be stored at a wholesale scale, electricity prices are highly volatile, with the existence of both positive and negative price peaks, and quite heavy tails. Thus, first MS models applied to electricity were for prices "alone": modelling of jumps, spikes, heavy tails and mean-reversion by Deng (1998), Ethier and T. Mount (1998), Huisman and Mahieu (2003), Janczura and Weron (2010). Other authors later studied the impact of exogenous variables on transition probabilities, using load and reserve margin as covariates (T. D. Mount, Ning, and X. Cai, 2006), or for example temperature (Huisman 2008). Zachmann (2013) used a MS model with exogenous variables to model the merit-order of the supply curve, using fuel prices and CO2 emissions. Finally, a demand-supply structure indicator was used by Kanamura and Ōhashi (2008) as an external regressor for both the price equation and the transition probabilities of a MS model.

The originality of this work is to use a MS model to disentangle the merit-order effect of RES production on electricity prices, depending on the state of the supply-demand equilibrium. Indeed, after applying an inverse hyperbolic sine transformation to the price time series (i.e. a log-like transformation that enables to take into account negative prices), we show that there are two distinct price regimes, with "high" and "low" prices. Then, we find as expected that the impact of RES production on the price is larger in periods of high prices than in periods of low prices. Moreover, we show that although electricity demand is the main driver for the regimes that are thus highly correlated with hours of the day but also weekdays and seasons, RES production plays a very important role as well, in particular by increasing the probability of switching to a low-price regime. Furthermore, we are able to distinguish the effect of wind power from the effect of solar production.

While their direct impact on the price is relatively the same, wind power influences much more the probabilities of transition than solar production.

In the next section, we briefly present the German power sector and the data we used, before showing some useful descriptive statistics. Then, we explain the inverse hyperbolic sine transformation applied to the price and the model itself in section 3. We show our results in section 4, along their interpretation. Finally, section 5 concludes the article and suggests some additional modelling possibilities that could be investigated.

2. DATA

2.1. The German power production sector

Studying the merit-order effect induced by RES production in Germany is quite relevant, since they have had a huge development of RES, especially wind and PV, for the past two decades (figure 2). Indeed, according to RAP (2015), the total installed capacity in Germany was 192GW in 2014, for a peak demand of 83GW and a gross electricity consumption of 576TWh, while the wind and solar capacity reached 76GW and produced 87.6TWh (slightly more than 15% of the total consumption). Beside RES generation, lignite and hard coal fired power plants are the main sources of electricity, with respectively 28GW and 21GW installed in 2014, for a production of 110TWh and 156TWh (19 and 27%). Nuclear and gas-fired power plants come next, but the former are being phased out, as established by the law, and the latter are less competitive than coal and lignite power plants at the moment. Also, Germany is interconnected capacity with Austria, Switzerland, the Czech Republic, Denmark, France, Luxembourg, the Netherlands, Poland, and Sweden, and is a net importer (36TWh in 2014), still according to RAP (2015).

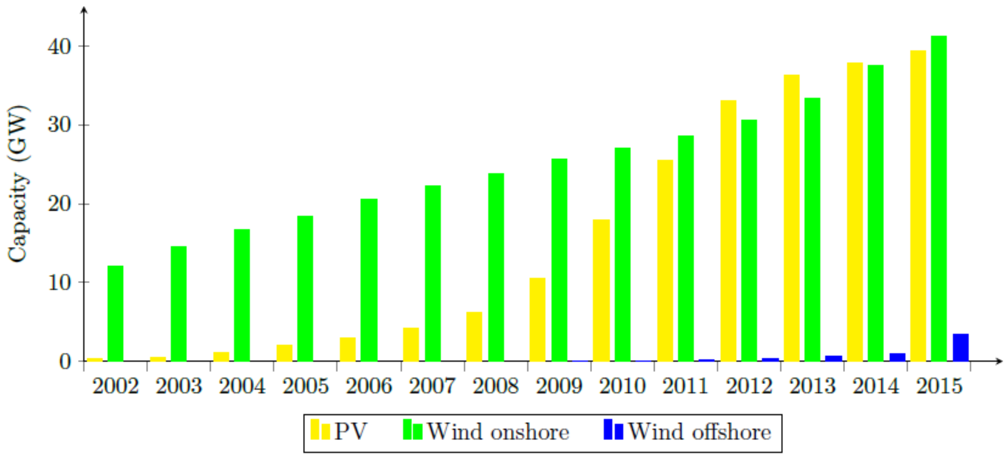


Figure 2. Cumulative installed wind and solar capacities in Germany (Source: Fraunhofer Institute)

Finally, studying the impact of RES production on electricity prices in Germany is quite relevant, since more than 40% of the electricity production in Germany (and Austria) is traded on the EEX day-ahead spot market, whose price is thus a good indicator of the electricity supply-demand equilibrium, compared to other European countries.

2.2. Overview of the data and correlations

Our data was originally composed of four time series for the period 2014-2015, that came from various sources:

- day-ahead electricity spot prices (from EEX-Powernext database) with an hourly time step;

- solar and wind electricity production, obtained from the four TSOs (TenneT, Amprion, 50Hertz, Transnet BW) websites, with a 15 minutes time step;
- hourly electricity load (from ENTSO-E website).

We summed the RES production of the TSOs and converted it to the hourly time step, in order to obtain 17,520 values for each variable. Figure 3 represents the corresponding final time series, and table 1 shows the correlation coefficients between the variables.

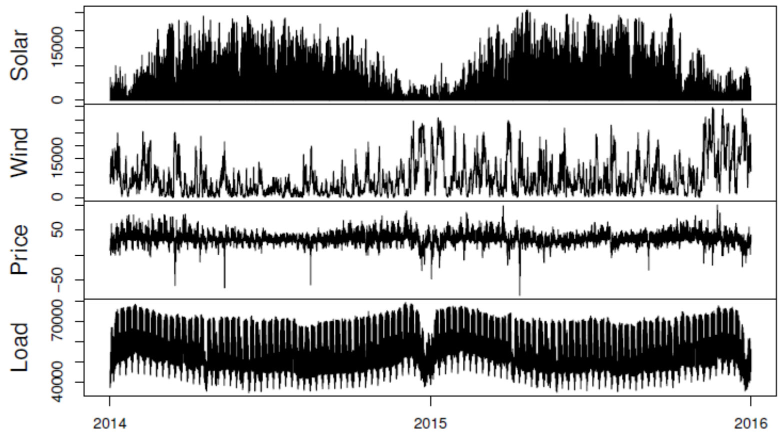


Figure 3. Final data

	Wind	Price	Load
Solar	-0.157	-0.039	0.331
Wind		-0.424	0.037
Price			0.674

Table 1. Pearson correlation coefficients between the variables

As one would expect, the solar production has a strong yearly seasonality, with much more production during summer, while wind production has a relatively opposite seasonality. Solar production also has a daily pattern (day/night), which is not observed for wind. However, wind and solar production are a bit negatively correlated ($\rho = -0.16$). The electricity demand is very cyclical as well, with annual and daily patterns, the latter being probably the reason for the positive correlation with solar production ($\rho = 0.331$). We also find these patterns in the price series, which is highly and positively correlated with load ($\rho = 0.67$), while it is negatively correlated with wind ($\rho = -0.42$), but almost not with solar production ($\rho = -0.04$). Finally, although the electricity consumption is higher in winter, essentially because of heating and lighting, there is a huge decrease in demand (and hence in price) during the Christmas holidays, because of a drop in the industrial activity (Do, Lin, and Molnár, 2016).

2.3. Descriptive statistics

Let us now present some descriptive statistics on the variables, before taking a closer look at the price and load statistics. Table 2 shows the main descriptive statistics for these variables. We notice that the price is quite volatile (C.V. = S.D./Mean = 0.39), has heavy tails ($\kappa = 6.2$) and is slightly skewed ($S = -0.3$), which are common features of electricity prices, and can be the result of the existence of a regime-switching mechanism, as explained by Krolzig (1997). As already seen in figure 3, there are negative prices (190 occurrences, i.e. a little more than 1% of the total), down to -79.9€/MWh, and positive spikes as well, up to 99.8€/MWh. These specificities of electricity prices are mostly due to the fact that electricity is almost non storable, so that demand must be met by production all the time, while production plants have flexibility constraints (limited ramp up/down). Any change in demand and/or production will then have an immediate impact on the price, that reflects the supply-demand equilibrium.

Compared to price, load is much less volatile (C.V. = 0.18). Furthermore, it has very flat tails ($\kappa = 1.8$) and is almost not skewed. Hence, despite the strong correlation between the two variables, these characteristics cannot explain the ones observed for the price, and it is therefore interesting to look at the RES production statistics. We observe that wind and solar productions are much more volatile and skewed than price (they have positive skewness, i.e. more high values than low ones, whereas the price is negatively skewed, i.e. has more low values than high ones) and have quite heavy tails as well. This suggests they might play a particular impact on the characteristics of the price. Finally, Jarque-Bera tests conducted on each time series indicate that they are all strongly non-normal (p-values < 2.2.10−16), which was expected from the descriptive statistics. The following subsections will now expose descriptive statistics of the variables as well as more in-depth statistical analyses of the price and load time series.

2.4. Price statistics

Since we are interested in modelling the electricity prices, we need to take a deep look at the price time series, which is the goal of this subsection. For simplicity of presentation, we chose to display a limited number of figures and tables within the text, while the rest of them is gathered in appendix A.

2.4.1. Distribution

The histogram (figure 4 hereinafter), the normal QQ-plot (figure 16) and the boxplot (figure 17) show strong evidence of non-normality, as already mentioned before. It is also interesting to look at the "detailed" histogram in figure 18, in order to reveal some peculiarities. Indeed, we can see that there are some isolated spikes, for example near zero, that probably correspond to marginal prices of some specific units (zero being the approximate marginal price of renewables). A closer look around zero shows that there is in fact a range of prices from approximately -0.10 to 0.10€/MWh (figure 19).

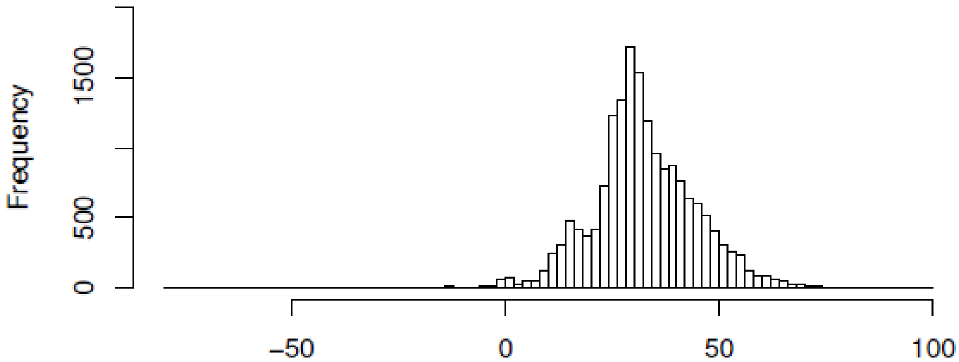


Figure 4. Histogram of price

2.4.2. Temporal analysis

From the plot of the autocorrelation and partial autocorrelation functions (figures 20 and 21), we see that the serial effect of the price series is very important. In particular, we notice high partial autocorrelations at the 1st, 2nd, 24th 25th lags (and lags number $25 + k \times 24$). These latter reflect the daily seasonality while the former show that there is a short-term serial dependence, which is an argument in favour of the existence of price regimes with a few hours of duration.

2.4.3. Unit root and stationarity tests

Finally, we make sure that the price time series is stationary by performing several unit root tests: augmented Dickey-Fuller, Phillips-Perron; and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) stationarity test (appendix A, table 6). All these tests show that the price time series is stationary.

	Min	Q1	Med	Mean	Q3	Max	S.D.	C.V.	Skewness	Kurtosis
Price (€/MWh)	-79.9	25.4	31.1	32.2	40.0	99.8	12.7	0.39	-0.3	6.2
Load (MW)	34,801	48,825	57,237	57,616	66,891	79,120	10,332	0.18	-0.02	1.8
Wind (MW)	28.8	2,668	5,590	7,811	10,845	34,662	6,890	0.88	1.3	4.2
Solar (MW)	0	0	140.8	3,866	6,314	25,812	5,857	1.52	1.5	4.2
Solar>0 (MW)	0.25	944	4,825	6,697	11,362	25,812	6,361	0.95	0.77	2.5

Table 2. Descriptive statistics

2.5. Load statistics

If we now look at the histogram of load (figure 5, left), we see that there appears to be two modes, with respectively "high" and "low" demand. When studying relations between price, RES production and demand, it is frequent to consider the residual (net) demand, i.e. demand minus RES production. The histogram of the residual load is visible on figure 5 right. We clearly see that the "high-demand mode" has disappeared, which can be explained by the fact that load and solar production are quite correlated because of their daily seasonalities. The descriptive statistics of the residual load are shown in table 3. We see that although it is still not Gaussian, there is almost no skewness nor (negative) excess kurtosis.

Finally, net demand has a correlation coefficient with price of 0.88, which makes it a "better" candidate for the model than the gross demand (ρ = 0.67). Similarly, it is often convenient to study the relative share of RES production, as in Ketterer (2014). In our case, they have higher correlation coefficients with price (-0.56 for wind and -0.095 for solar) than RES production itself (-0.42 and -0.049), but also with residual load (-0.62 and -0.18). This could lead to high variance inflation factors, but because of the strong seasonality of load and price (see subsection 2.6 below), it is necessary to control the price with a load variable, and as we will see in section 4, the coefficients will still be very stable.

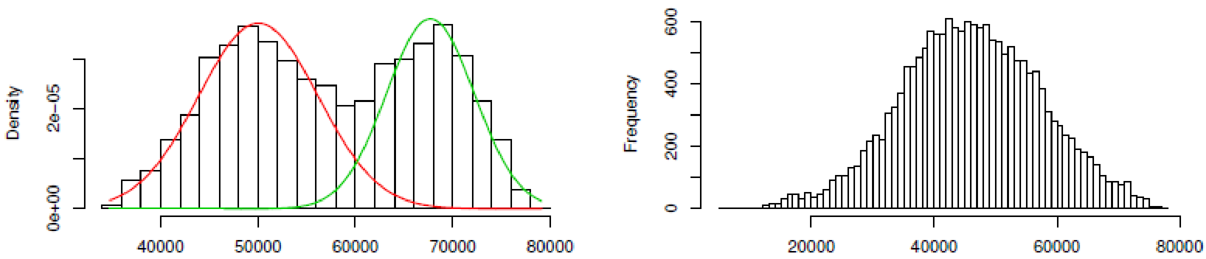


Figure 5. Histogram and estimated mixture model densities of load (left), and histogram of residual load (right)

	Min	Q25	Median	Mean	Q75
Residual load (MW)	5, 903	38, 214	45, 845	45, 939.9	53, 905
	Max	S.D.	C.V.	Skewness	Kurtosis
Residual load (MW)	77, 898	11, 425	0.2486878	-0.1	2.9

Table 3. Descriptive statistics of the residual load

2.6. Price and load seasonality

Electricity prices are highly seasonal, mainly because of the demand seasonality. Figure 22 in appendix B shows the power spectra (or periodograms) of the price, load and residual load time series. These enable to detect the dominant frequencies in the series. We find that the three main periods (computed as the inverse of the frequencies) are 12h, 24h, and 168h (a week), followed by 84h (half a week), 3000h (approximately a third of a year), and 9000h (approximately a year). Although the amplitudes and relative importance associated to each period are different for each time series, we can definitely tell from these values that price and load have pretty much the same seasonalities, which is comforting. Hence, including electricity demand in the model will help control the seasonality the electricity prices.

3. MODEL

3.1. Variable transformation

3.1.1. Problems associated with logarithmic transformation

It is common in financial time series analysis to consider the series in logarithmic form, because it has interesting properties, from reducing non-normality to being able to interpret coefficients of regressions as elasticities, or in order to interpret first differences of the transformed data as rate of returns.

However, we cannot take the logarithm of the price, because of the negative prices. This could be artificially prevented by adding an offset value to the series so that it would be strictly positive, enabling then to use the log transformation, as suggested by Sewalt and De Jong (2003). However, the log transformation is not even well suited for (positive) electricity prices, because there can be very high prices for very short periods, and the log would neglect the impact of those very high prices. Hence, adding a relatively high arbitrary value to the price before taking the log would just increase this undesirable effect. If the threshold was low enough compared to the mean, it would produce little distortion, but in our case, the price goes as low as -79.9€/MWh, for a mean of 32.2€/MWh and a maximum of 99.8€/MWh. Adding 80€/MWh to prices to have only strictly positive values would then highly “compress” the high prices together while maybe giving “too much” importance to the negative prices.

3.1.2. Inverse hyperbolic sine transformation

In order to mitigate this effect while taking into account negative prices we use an inverse hyperbolic sine transformation, as suggested by Johnson (1949) and more recently by Schneider (2012) for electricity prices. This transformation is achieved by applying the inverse of the hyperbolic sine function, which is defined by the following expression, with offset and scale parameters ξ, λ ∈ R:

$$\forall x \in \mathbb{R} \quad f(x,\xi,\lambda) = \sinh^{-1}\left(\frac{x-\xi}{\lambda}\right) = \ln\left[\frac{x-\xi}{\lambda} + \sqrt{\left(\frac{x-\xi}{\lambda}\right)^2 + 1}\right]$$

The behaviour of this function is logarithmic when $|x| \rightarrow +\infty$ and linear when $x \rightarrow \xi$:

$$f(x,\xi,\lambda) \underset{x\to\xi}{\sim} \frac{x-\xi}{\lambda}$$

$$f(x,\xi,\lambda) \underset{|x|\to+\infty}{\sim} \operatorname{sign}(x) \times \ln\left(\frac{2|x|}{\lambda}\right) = \operatorname{sign}(x) \times \left[\ln\left(\frac{|x|}{\lambda}\right) + \ln 2\right]$$

Figure 6 below represents the inverse hyperbolic sine function with ξ = 0 and λ = 1 in red, and in green the symmetric logarithmic function (with position factor ln 2) and its symmetric with respect to the origin.

We decided to take the mean value of the price for ξ in order to separate higher from lower prices, according to the expectation of differentiated merit-order effects for high and low prices, and we choose λ = 1MWh for simplicity reasons. The histogram of the transformed price (figure 7) lets appear two distinct modes that could not be seen in the original histogram. This is due to the fact that the logarithmic behaviour of the inverse hyperbolic sine function “compresses” the extreme values, while the linear behaviour “dilates” them. However, what could seem artificial at this stage will prove very useful for the estimation. Also, this particular feature is another argument in favour of a switching mechanism that are known to create multimodal distributions. Fitting this distribution as a mixture of two normal distributions gives the red and green densities in figure 7. The two modes are well identified, but are still not Gaussian (and they have no reason to be).

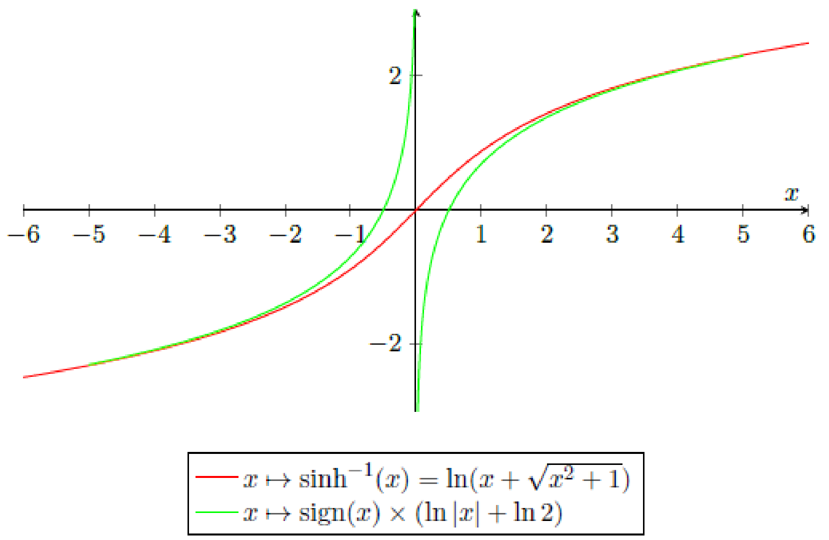


Figure 6. Inverse hyperbolic sine (red) and symmetric logarithm (green) functions

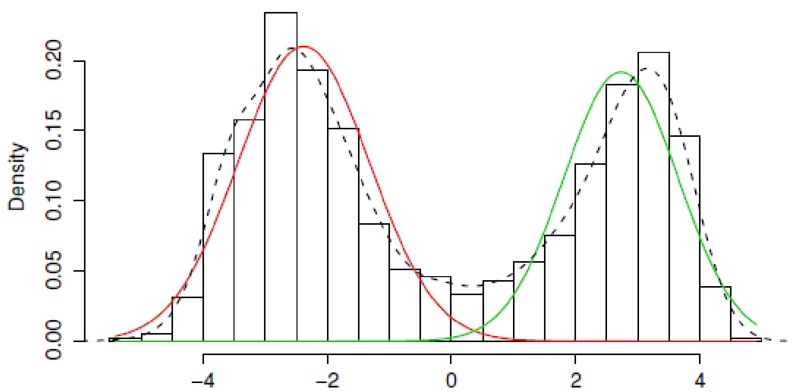


Figure 7. Histogram, estimated mixture model densities and kernel density (dashed) of the transformed price

3.2. Model

Now that we have described and transformed the data, we develop an econometric model to quantitatively evaluate the impact of RES generation on electricity prices. We consider a discrete-time ($t \in \mathbb{N}$), two-regime MS model, in which the coefficients of the covariates and the variance of the residuals depend on the value of a latent unobserved state (or regime) variable $S \in \{1; 2\}$. The state variable is a Markov chain, i.e. the probability of switching from a regime is Markovian (it only depends on its current state, and not its past):

$$\forall t \in \mathbb{N} \quad \mathbb{P}(S_t = j | S_{t-1} = i, S_{t-2}, ..., S_0) = \mathbb{P}(S_t = j | S_{t-1} = i) = p_{ij}(t) \qquad (1)$$

and we denote by P the stochastic matrix associated with the process:

$$\forall t \in \mathbb{N} \quad P(t) = \begin{pmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{pmatrix} \quad \text{with:} \quad \forall i \in \{1,2\} \quad p_{i1} + p_{i2} = 1$$

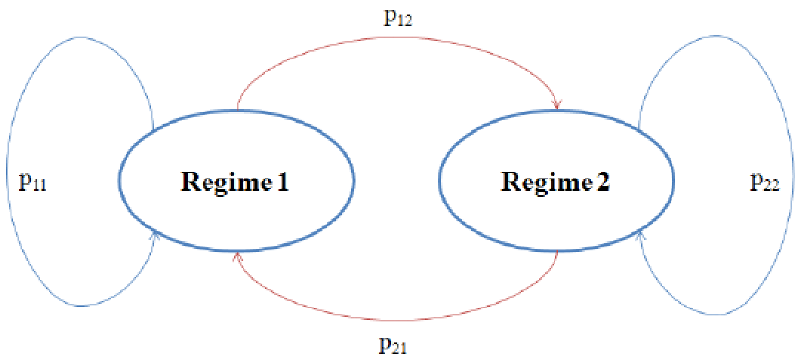


Figure 8. Simplified view of the Markov switching model

The principle of the MS model is shown in a simplified manner in figure 8, while the model to estimate is given by equations 2-4 below (note that all the parameters of the model are estimated together):

$$\forall t \in \mathbb{N} \quad Price_t^* = \beta_0(t) + \beta_1(t)\frac{Wind_t}{Load_t}\beta_2(t)\frac{Solar_t}{Load_t} + \beta_3(t)RLoad_t + \varepsilon_t \qquad (2)$$

where RLoad = Load − Wind − Solar is the residual load, Price* is the transformed price, and:

$$\forall t \in \mathbb{N} \quad \begin{cases} \beta(t) = \beta^{(1)} \times \mathbb{1}(S_t = 1) + \beta^{(2)} \times \mathbb{1}(S_t = 2) \in \mathbb{R}^4 \\ \varepsilon_t \rightsquigarrow \mathcal{N}(0, (\sigma_1)^2 \times \mathbb{1}(S_t = 1) + (\sigma_2)^2 \times \mathbb{1}(S_t = 2)) \end{cases} \qquad (3)$$

Furthermore, we allow the Markov chain to be inhomogeneous, i.e. the probabilities of transition can vary over time. One of the simplest and widely spread specification for inhomogeneous MS models is to have the probabilities being described by a logistic function of external regressors. In our case, we want probabilities to vary with wind penetration, with the following logistic link (to be estimated as well):

$$\forall t \in \mathbb{N} \quad \text{logit}(p_{i1}(t)) = \ln\left(\frac{p_{i1}(t)}{1 - p_{i1}(t)}\right) = \alpha_0^{(i)} + \alpha_1^{(i)}\frac{Wind_t}{Load_t} + \alpha_2^{(i)}\frac{Solar_t}{Load_t} \qquad (4)$$

$$\Leftrightarrow \quad p_{i1}(t) = \frac{1}{1 + \exp\left(-\alpha_0^{(i)} - \alpha_1^{(i)}\frac{Wind_t}{Load_t} - \alpha_2^{(i)}\frac{Solar_t}{Load_t}\right)}$$

$$\Leftrightarrow \quad p_{i2}(t) = 1 - p_{i1}(t) = \frac{1}{1 + \exp\left(\alpha_0^{(i)} + \alpha_1^{(i)}\frac{Wind_t}{Load_t} + \alpha_2^{(i)}\frac{Solar_t}{Load_t}\right)}$$

It is also possible to compute the expected duration of each regime. For constant probabilities, the expected duration has the following simplified expression:

$$\mathbb{E}\tau_i = p_{ij}\sum_{k=0}^{+\infty} k p_{ii}^k = \frac{1}{p_{ij}} = \frac{1}{1 - p_{ii}}$$

Similarly, still for a homogeneous MS model, one can compute the (long-term) stationary distribution π = (π1, π2), which gives the mean proportion of each regime, i.e. such that:

$$\lim_{n\to+\infty} P^n \underset{not.}{=} P^\infty = \begin{pmatrix} \pi_1 & \pi_2 \\ \pi_1 & \pi_2 \end{pmatrix}$$

and which also verifies:

$$\pi P = \pi \Leftrightarrow \pi_1 = 1 - \pi_2 = \frac{p_{21}}{p_{12} + p_{21}} = \frac{\mathbb{E}\tau_1}{\mathbb{E}\tau_1 + \mathbb{E}\tau_2}$$

For an inhomogeneous MS model, these values have usually no analytical expression, but can nevertheless be estimated numerically.

4. RESULTS AND INTERPRETATION

4.1. Results

The model is estimated by conditional maximum likelihood under EViews 9.5, using the Broyden-Fletcher-Goldfarb-Shannon (BFGS) algorithm, with Marquardt step method. We use the full sample (T = 17, 520). We tested the robustness of the estimation by removing the first and last four days from the sample (192 hours in total, i.e. roughly 1% of the whole sample), and we found that the estimates were very little affected by this operation, which is comforting.

The results of the estimation are given in table 4. We first notice that all coefficients are highly significant (*p-values* < 10−4). Also, one of the main questions raised by MS models is the number of switching coefficients used. To make sure there is a significant switching effect, we perform Wald tests on each parameter, to make sure that the coefficients are significantly different from a regime to another. The probabilities of transition p11 and p21 (and thus also p12 and p22) are also significantly different. This means that there is indeed a Markovian structure and hence serial correlation within and between each regime, at the opposite of a 'simple switching' model, for which the probability of being in each regime is unconditional.

Both $\alpha_1^{(i)}$ and $\alpha_2^{(i)}$ are positive, which means that the probability of staying in or switching to regime 1 (resp. 2) is an increasing (resp. decreasing) function of the relative shares of wind and solar. Figure 9 shows the evolution of the modelled probabilities of transition with the relative shares of wind and solar. This graph reveals two particular features: firstly, the influence of the wind production is much stronger than the influence of the solar one; secondly, the impact on p_{11} (and hence on p_{12}) is highly limited, compared to the impact on p_{21} (and p_{22}).

4.2. Regime visualisation and interpretation

The results in table 4 are interesting, but we would like to associate the regimes with periods of time, and interpret the values taken by the variables during these states. Fortunately, the estimation procedure computes for each time step the probability of being in each regime. In fact three kind of probabilities are computed by the algorithm:

- one-step probabilities: $P(S_t = i | S_{t-1})$;
- filtered probabilities: $P(S_t = i | S_{t-1} \ldots S_1)$;
- smoothed probabilities: $P(S_t = i | S_1 \ldots S_T)$.

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Regime 1				
Intercept	−7.171284	0.056058	−127.9269	0.0000
Wind/Load	1.271735	0.064171	19.81791	0.0000
Solar/Load	1.530959	0.064736	23.64942	0.0000
RLoad	0.000114	$1.19e-6$	95.71098	0.0000
σ_1	0.581968	0.006004	96.93304	0.0000
Regime 2				
Intercept	−3.416402	0.089319	−38.24955	0.0000
Wind/Load	1.569330	0.135748	11.56058	0.0000
Solar/Load	2.242196	0.126642	17.70505	0.0000
RLoad	0.000106	$1.40e-6$	75.80216	0.0000
σ_2	0.695252	0.008222	84.56472	0.0000
Transition Matrix Parameters				
$\alpha_0^{(1)}$	1.109705	0.059471	18.65958	0.0000
$\alpha_1^{(1)}$	5.306012	0.345166	15.37235	0.0000
$\alpha_2^{(1)}$	4.464573	0.414828	10.76247	0.0000
$\alpha_0^{(2)}$	−2.762521	0.069484	−39.75780	0.0000
$\alpha_1^{(2)}$	6.971012	0.392717	17.75075	0.0000
$\alpha_2^{(2)}$	1.483876	0.441446	3.361400	0.0008

Time-varying transition probabilities:
$p_{ij}(t) = \mathbb{P}(S_t = j | S_{t-1} = i)$

		1	2
Mean	1	0.876859	0.123141
	2	0.182783	0.817217

		1	2
Std. Dev.	1	0.060346	0.060346
	2	0.150465	0.150465

Time-varying expected durations:

	Regime 1	Regime 2
Mean	12.64588	8.048185
Std. Dev.	13.72816	3.877061

Table 4. Estimation results of the inhomogeneous MS model

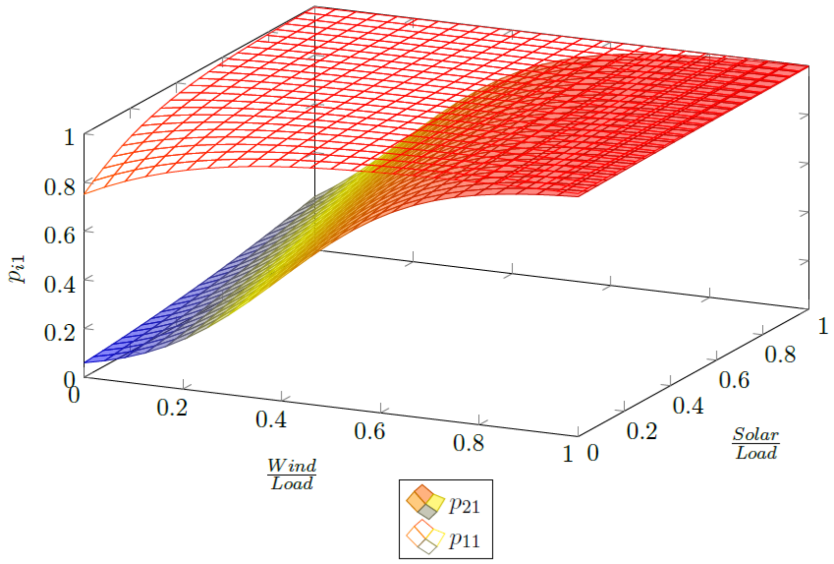


Figure 9. Probabilities of transition

Filtered or smoothed probabilities are often used to attribute the regimes to each time step. We will use smoothed probabilities, but there is almost no difference with the filtered probabilities in the regime attribution. In practice, the probabilities are very close to 1 or 0, so that the choice between a regime or another is unambiguous. A very common and useful way to visualise the switching process is shown in figure 10. From this short sample, we clearly see that as expected, the regimes are associated to a price mode: prices are higher in regime 2 than in regime 1. Also, these regimes seem to be quite correlated to the hours of the day. This is normal since prices have a strong seasonality, as already explained in subsection 2.6. However, there are some exceptions, which the MS model is able to take into account, as we can see in this example. Furthermore, the MS model is much more parsimonious than a model with for example hourly dummies. And above all, it manages to give the desired differentiated merit-order effect.

These graphs contain a lot of information that we will now look into. First of all, we can notice the very regular pattern followed by the load, with a peak at 11am and very low demand during the night and the weekend. The first two days wind and solar productions are rather low, and the regimes seem to be driven by load only: regime 2 coincides with high demand and high prices, from 6am to 10pm, so basically during the day. On Wednesday and Thursday however, wind and solar production increase, which incurs a change in regime with a decrease in price, respectively from 12 to 5pm and from 12 to 3pm. On Friday, wind production is rather low, but solar production reaches its week peak at 1pm, and there is a slight change of regime (P(S = 1) = 0.66). Finally, the load decrease during the weekend, associated with a strong solar production, keeps the prices low.

Figure 11 shows the frequency of being in regime 1 for each hour. We clearly see that night hours are highly correlated with the low-price regime, while peak hours such as 8-10am and 6-10pm are more associated with high prices, but with a lower frequency. The middle of the day does not belong to a specific regime, with frequencies between 40% and 60%. In fact, the frequency differs a lot among months, reaching 88.7% at 1-2pm in March while it is of only 41.7% in November. This can be explained by the fact that the average wind production is of 11GW in November while it is of 8GW in March. Despite an overall equivalent total RES production, the impact of wind production on the price regime is central through its influence on the probabilities of transition, as already seen in figure 9.

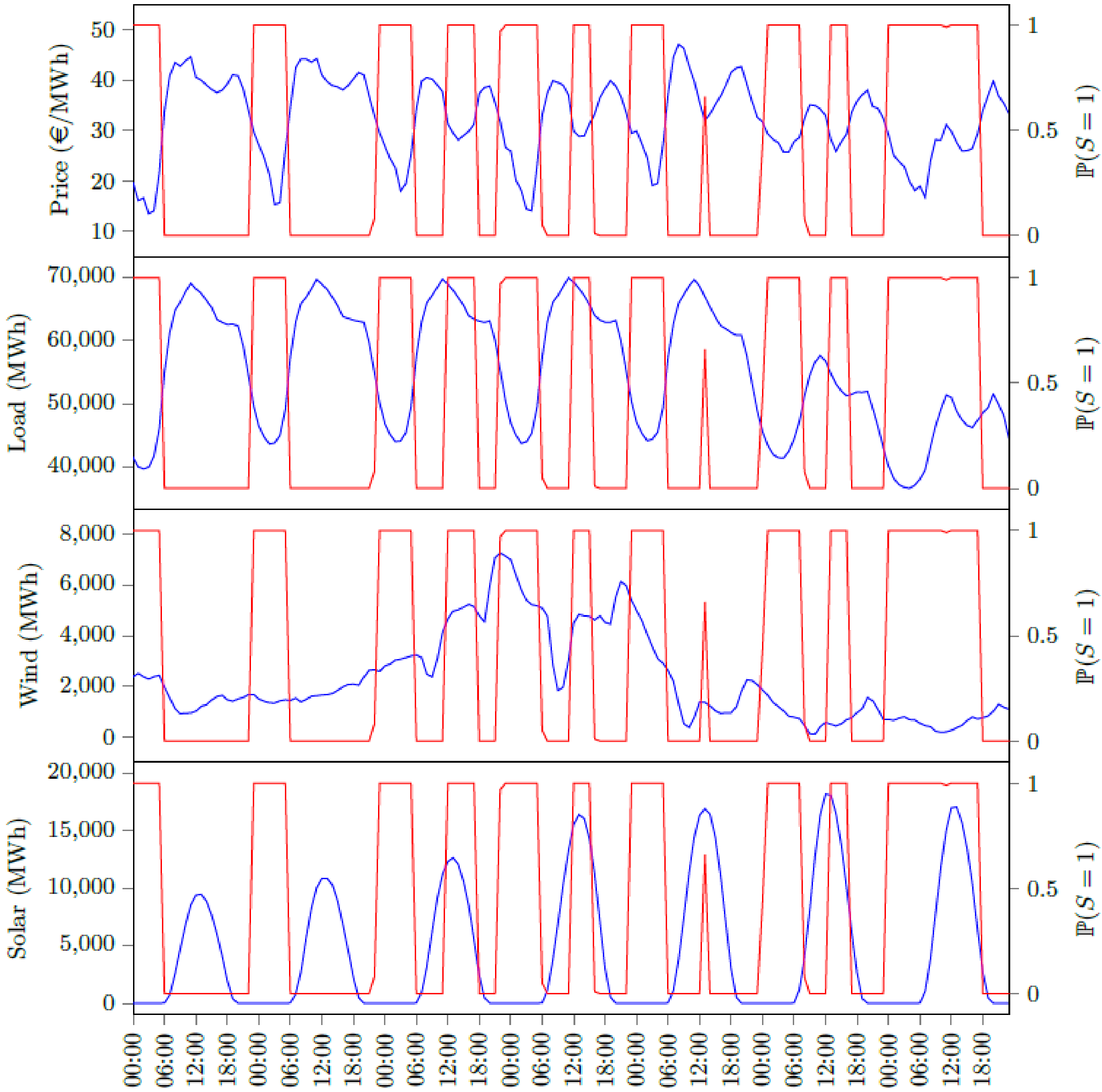


Figure 10. Price, gross demand, wind and solar productions (blue, left axis) and probability of being in regime 1 (red, right axis), first week of September 2014

We can also visualise the regimes thanks to 2D plots such as figure 12: (transformed) price vs. (logarithm of) load. On this graph the modes are easily identifiable, and we have the impression that the slope is higher in regime 2 (red) than in regime 1 (blue). Also, this partly confirms what we saw in figure 10, i.e. that price regimes are highly correlated with load regimes. When looking at the histograms of the transformed price for each regime (figure 13), we find that the two modes correspond to the previously identified ones in figure 7. However, we can see from the Price-Load graph that there exists a region for which load is high, but the price remains low. This is confirmed by taking a look at the histograms of load for each regime, as we can see in figure 14. For the transformed price, the two modes were clearly identified, whereas only a high-demand mode is well identified for load, while low prices can occur with both high and low demand (the latter being more frequent). However, if we plot the transformed price versus the residual demand (which was used as a covariate in the model), this region disappears (figure 15).

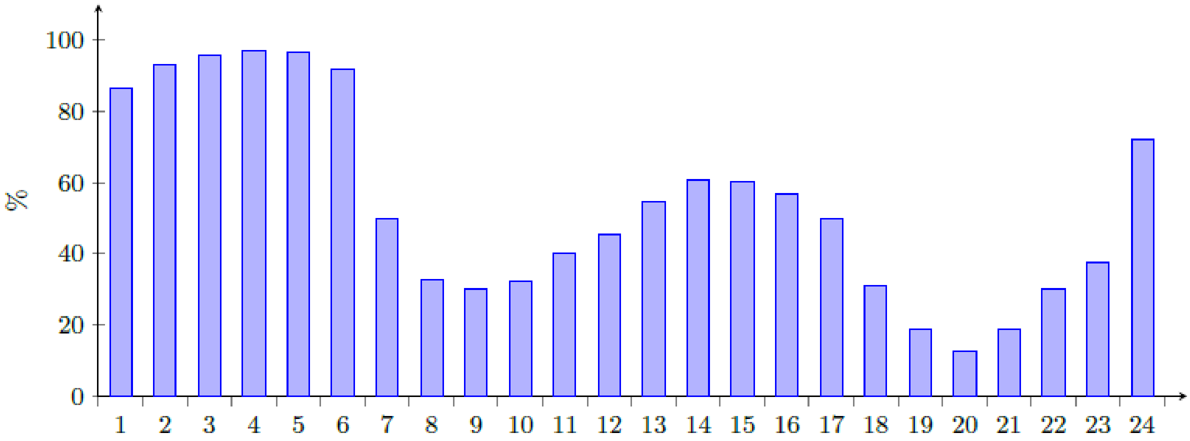


Figure 11. Mean frequency of occurrence of regime 1 for each hour of the day

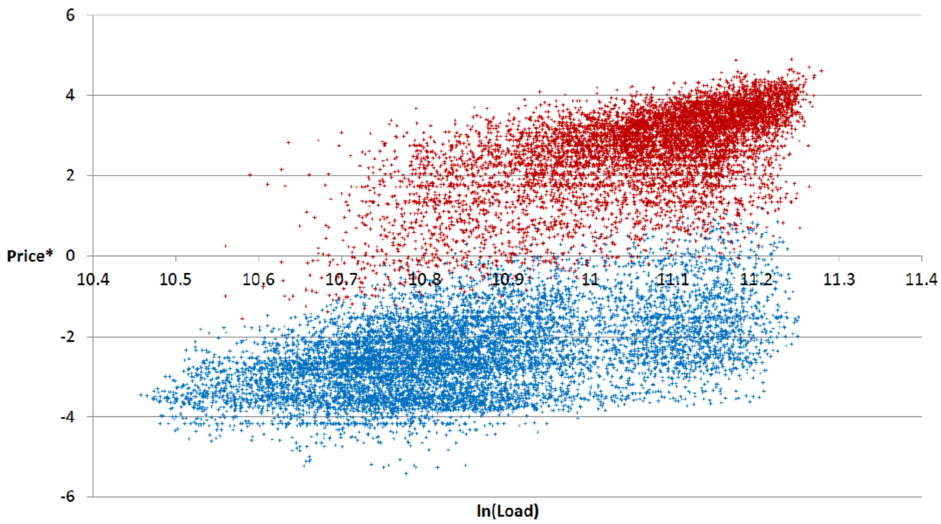


Figure 12. Price* vs. ln(Load) for the two regimes (blue: regime 1, red: regime 2)

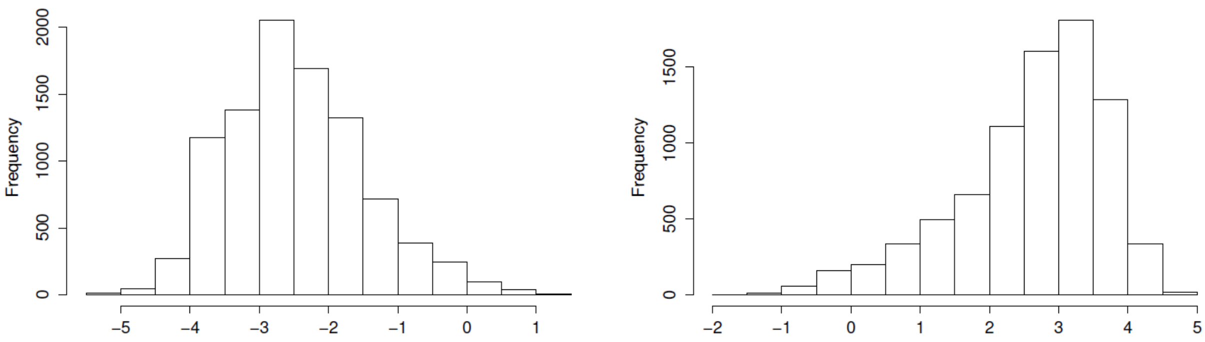


Figure 13. Histograms of the transformed price for regime 1 (left) and 2 (right)

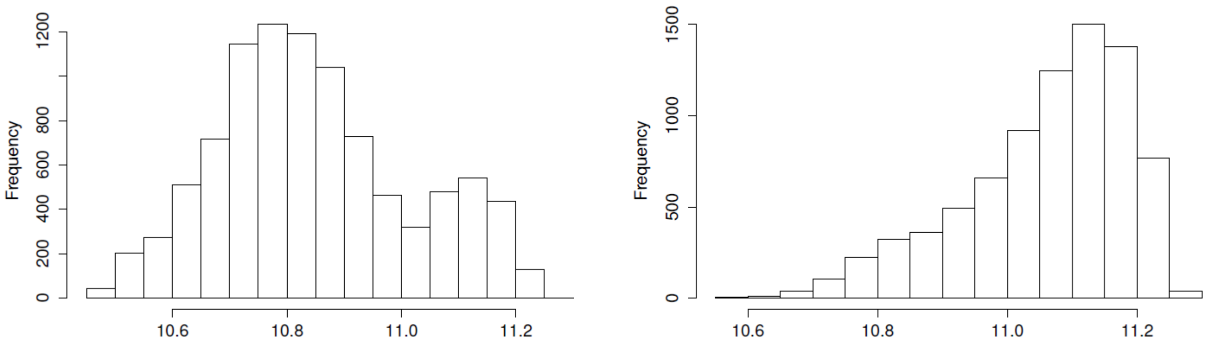


Figure 14. Histograms of the logarithm of load for regime 1 (left) and 2 (right)

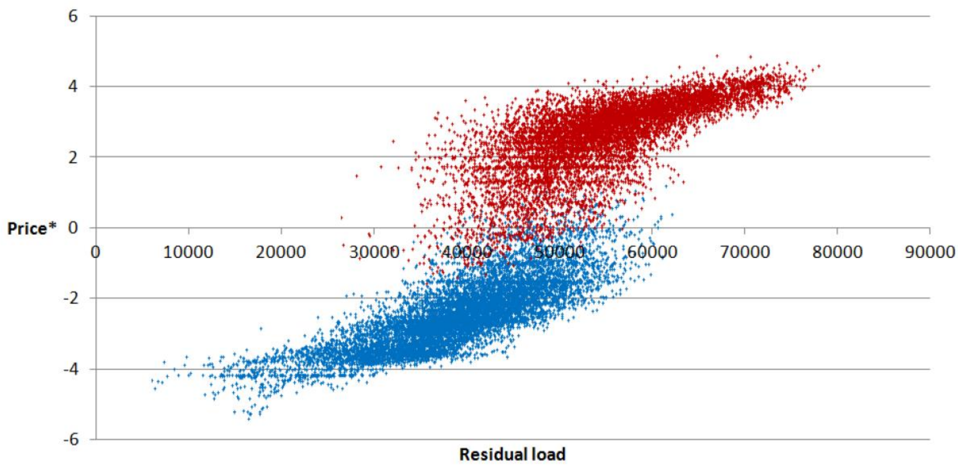


Figure 15. Transformed price vs. residual load in regime 1 (blue) and 2 (red)

4.3. Marginal effects

Interpreting the coefficients can be difficult, since the price has been transformed and because we use relative values such as the shares of wind and solar production as well as absolute residual load. Also, the coefficients associated with wind and solar productions are positive, which would mean, without looking at the residual load, that they increase the price. However, when taking into account the effect of RES through the residual load coefficient we should find a negative effect. Hence, it is interesting to derive an expression for the marginal effects (slopes) that would tell how much the decrease in price is when RES production and load increase. We do not compute elasticities, because they would be very high when prices approach zero, and would change sign whenever the price does. For clarity reasons, we put in appendix C the computations and the formal expressions of the marginal effects (equations 8-10), while we show the results here (table 5)[4].

As expected, we find negative marginal effects for RES production, and a positive marginal effect for load, for each time step. On average, an increase of 1GW of wind will decrease the price in regime 1 (resp. 2) by 0.77€/MWh (resp. 1€/MWh). The influence of solar is slightly weaker, as an extra gigawatt hour lowers the price of 0.73€/MWh in period 1, and 0.96€/MWh in regime 2. On the contrary, if the demand increases by 1GW in regime 1 (resp. 2), the price increases on average by

[4] The standard deviations are computed directly from the time series generated by equations 8-10. To obtain robust standard deviations, one should use bootstrap methods in order to take into account the variance of the estimators.

0.93€/MWh (resp. 1.18€/MWh). We performed Welch t-tests on the marginal effects to make sure their means are significantly different from one another (from a regime to another and for all variables for each regime). Finally, note that these marginal effects are only valid inside each regime when there is no switching. Indeed, when there is a regime change, marginal effects are not defined since there is a non-differentiability. For example, the predicted change of the transformed price conditionally on the previous period would be:

$$\mathbb{E}\left[\Delta Price^{*}|S_{t-1}=i\right] = p_{ii}\left[\beta_1^{(i)}\Delta\left(\frac{Wind}{Load}\right)+\beta_2^{(i)}\Delta\left(\frac{Solar}{Load}\right)+\beta_3^{(i)}\Delta RLoad\right] + p_{ij}\left[\Delta\beta_0+\Delta\left(\beta_1\frac{Wind}{Load}\right)+\Delta\left(\beta_2\frac{Solar}{Load}\right)+\Delta\left(\beta_3 RLoad\right)\right]$$

which is the sum of a regime *i* term and a regime-switching switching term. The first term is the equivalent of the marginal effect that is computed for an infinitesimal change of the covariates. This cannot be done with the second term, because of the switching coefficients.

This analysis confirms the existence of different merit-order effects in high-price and low-price regimes. However, these slopes have very high coefficients of variation, which means that inside each regime the marginal effect can vary a lot. Nevertheless, these high variations are partly due to the reverse transformation that is needed in order to compute those slopes, and the estimated coefficients β are on the contrary very well determined, with rather low standard deviations. It is thus important to remember that these slopes are not equivalent to the model, in which we use a non-linear transformation of the price, but only illustrate it in a linear way. In addition, when we estimate the MS model for the non-transformed price, we fail to disentangle low-price from high-price regimes, which confirms the usefulness of the price transformation.

	Regime 1			Regime 2		
	Mean	S.D.	C.V.	Mean	S.D.	C.V.
$\frac{\partial Price}{\partial Wind}$ (10^{-3}€/h)	-0.77	0.73	0.94	-1.0	0.79	0.79
$\frac{\partial Price}{\partial Solar}$ (10^{-3}€/h)	-0.73	0.68	0.94	-0.96	0.76	0.79
$\frac{\partial Price}{\partial Load}$ (10^{-3}€/h)	0.93	0.84	0.90	1.18	0.92	0.78

Table 5. Marginal effects for each regime

However, the existence of different marginal effects for high and low prices suggests that the inverse supply curves are on average convex, and that they could be approximated by a two-piece affine function. Indeed, if D: q ↦D(q) (with (D′ < 0) and S: q ↦ S(q) (with S′ > 0) are the expressions of the instantaneous demand and supply functions, the marginal effect of RES production and load in a domain of differentiability (which would be one of our two regimes) would be at the first order:

$$\frac{\partial p}{\partial RES} = -\frac{\partial p}{\partial Load} = \frac{1}{\frac{\partial D}{\partial p}-\frac{\partial S}{\partial p}} < 0$$

From this equation, it is clear that the higher the slope of the inverse supply function S^{-1}, the lower the slope of S (its inverse), and thus the higher the marginal effect. The fact that the marginal effects are not the same for wind and solar production (nor the strict opposite for the load) could be explained by the variability of both RES production and load, coupled to the fact that both the slopes of the supply and demand curves change over time. A limit to this approach is that the volume traded on the spot market represents only a fraction of the total electricity consumption, and the demand curve is thus not very representative of the real demand, which is in fact almost inelastic in the short run. The differentiation could also come from a breaking point in the demand curve, or from both. Anyway, a thorough analysis of the bidding curves would be required to confirm or invalidate these intuitions. Likewise, the volatility was found to be higher in regime 2 than in regime 1. This is consistent with the possibility that the slope of the inverse supply curve is higher in this regime: all other things being equal, the volatility incurred by supply and demand volatility is necessarily higher, for the main reason as before.

Another topic is the interpretation of the coefficients of the time-varying probabilities. We already explained the effect of the relative shares of RES on the probabilities through the sign of the coefficients, but it is also of interest to assess the impact on more understandable variables like the expected duration of each regime and the stationary distribution, as defined at the end of section 3. We presented analytical expressions for these variables in the homogeneous case that are no longer valid in the inhomogeneous one. However, deriving analytical expressions for the marginal effects in the inhomogeneous case can give an idea, albeit not totally correct, of the magnitude of the impact. Since the expression of the stationary distribution involves both $\alpha^{(i)}$, we will focus on the expected durations only, and on RES production (similar expressions can be derived for the load). We show in appendix C the following results:

$$\frac{\partial\left(\mathbb{E}\tau_1-1\right)}{\mathbb{E}\tau_1-1} \Big/ \partial Wind = \frac{\alpha_1^{(1)}}{\langle Load\rangle} \simeq 9.2\ \%.\mathrm{GW}^{-1} \quad ; \quad \frac{\partial\left(\mathbb{E}\tau_1-1\right)}{\mathbb{E}\tau_1-1} \Big/ \partial Solar = \frac{\alpha_2^{(1)}}{\langle Load\rangle} \simeq 7.7\ \%.\mathrm{GW}^{-1} \tag{6}$$

$$\frac{\partial\left(\mathbb{E}\tau_2-1\right)}{\mathbb{E}\tau_2-1} \Big/ \partial Wind = \frac{-\alpha_1^{(2)}}{\langle Load\rangle} \simeq -12\ \%.\mathrm{GW}^{-1} \quad ; \quad \frac{\partial\left(\mathbb{E}\tau_2-1\right)}{\mathbb{E}\tau_2-1} \Big/ \partial Solar = \frac{-\alpha_2^{(2)}}{\langle Load\rangle} \simeq -2.6\ \%.\mathrm{GW}^{-1} \tag{7}$$

i.e. $\alpha_1^{(i)}/\langle Load\rangle$ and $\alpha_2^{(i)}/\langle Load\rangle$ can be interpreted as the (absolute values of the) semi-elasticities of $E\tau i - 1$ with respect to wind and solar productions, respectively: on average, when there is an additional GW of wind (resp. solar), $E\tau_1 - 1$ increases by 9.2% (resp. 7.7%), while $E\tau_2 - 1$ decreases by 12% (resp. 2.6%). As a reminder, the computed mean expected durations where $E\tau_1$ = 12.6 h and $E\tau_2$ = 8.04 h. The semi-elasticities are quite strong, and it must be remembered that they are only approximated mean values, and only valid locally. However, these values confirm that RES production strongly affect the regimes probabilities, and thus their expected duration.

5. CONCLUSION

We developed a two-regime Markov switching model to assess the impact of variable renewable production (wind and photovoltaic) on German electricity spot prices. We found that under an appropriate inverse hyperbolic sine transformation, the price time series was in fact bimodal, with a "low-price" and a "high-price" regime. These two modes were very well identified by the estimation process, and the impact of the relative shares of both wind and solar production were found to be significantly different for each regime. Moreover, the results are in line with standard electricity markets theory: the higher the prices, the higher the merit-order effect, and the higher the volatility as well. To illustrate this, we computed the mean marginal effects of RES production and load. On average, an increase of 1GW of wind will decrease the price in regime 1 (resp. 2) by 0.77€/MWh (resp. 1€/MWh). The influence of solar is slightly weaker, as an extra gigawatt lowers the price of 0.73€/MWh in period 1, and 0.96€/MWh in regime 2. On the contrary, if the demand increases by 1GW in regime 1 (resp. 2), the price increases on average by 0.93€/MWh (resp. 1.18€/MWh). Although we made sure these marginal effects are significantly different from one another, they are much more variable than the estimated coefficients of the model. Also, note that these marginal effects are only valid inside each regime when there is no switching.

In addition, albeit the regimes are partly deterministic, because of the correlation with the (highly seasonal) load, there are many exceptions, i.e. low prices with high load. This is due to the RES production, whose influence can be measured in particular through its impact on the transition probabilities, but while the impact of solar and wind productions on the price is relatively the same, the effect on the probabilities is mostly due to wind. From the estimation of the transition probabilities, we also derived approximations of semi-elasticities for the expected duration of each regime. Overall, the endogenous determination of the regimes is an interesting feature of this model, which makes it quite parsimonious compared to a semi-deterministic model with hourly, daily, and seasonal dummies. To conclude this article, we will suggest in the following paragraphs some additional modelling possibilities that would enrich the subject.

First of all, although we only computed the effect of RES generation on the electricity price, there is necessarily an impact on the other means of production that have to adapt their production to meet the residual demand. Assessing this effect would require production data from plants such as hard coal, lignite, gas, and oil fired power plants, as well as nuclear power plants. Nevertheless, we can also expect to have a differentiated effect on these conventional power plants, depending on the merit-order and on which plant is marginal: on the one hand, in periods of high demand, high prices are caused by the need of expensive means of production to meet the demand. In the short run, only these would be impacted. On the other hand, when demand is low, coal and lignite power plants are the ones susceptible to reduce their production, as they are likely to be the marginal plants. However, these reflexions are only speculations, and a more thorough analysis would be needed to numerically evaluate this impact. Linking all these phenomena would require estimating a Markov switching vector model using thermal plants production as dependent variables, but also cross-border flows for example, while keeping RES production and load as exogenous variables (covariates). Unfortunately, the estimation of such models can be very difficult to achieve, and even more with time-varying probabilities. Hopefully, a univariate analysis could give a good approximation, and above all it would be easier to estimate. Otherwise, other approaches can be used to estimate the impact of RES generation on thermal production, as did Graf and Marcantonini (2016), who used panel data in Italy to show that RES production reduces CO_2 emissions while increasing the average plant emission factor.

Another research avenue would be the modelling of the volatility. Indeed, we only showed that volatility was higher during the high-price regime, which was already expected. Nonetheless, it is likely that intermittent RES production influences this volatility as well. This has been shown using GARCH modelling by Ketterer (2014) and Karanfil and Li (Forthcoming, 2017), which we cited in the literature review. In addition, Cifter (2013) modelled electricity prices in the Nordic power market with a MS-GARCH model, and showed that there were indeed two volatility regimes. However, this study did not include exogenous variables such as RES production in the variance equation. Hence, it could be very interesting to estimate a Markov switching GARCH model in order to determine whether and how RES production impacts the price volatility and the volatility regimes.

A. Price statistics

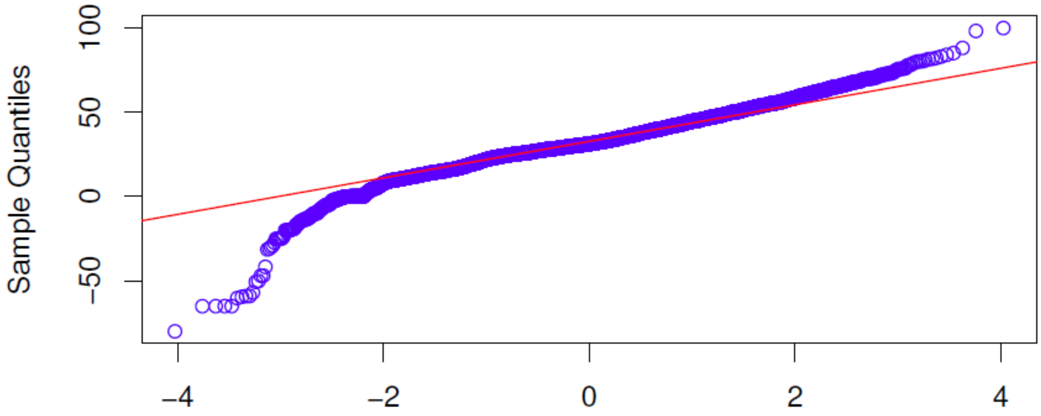


Figure 16. Normal QQ-plot of price

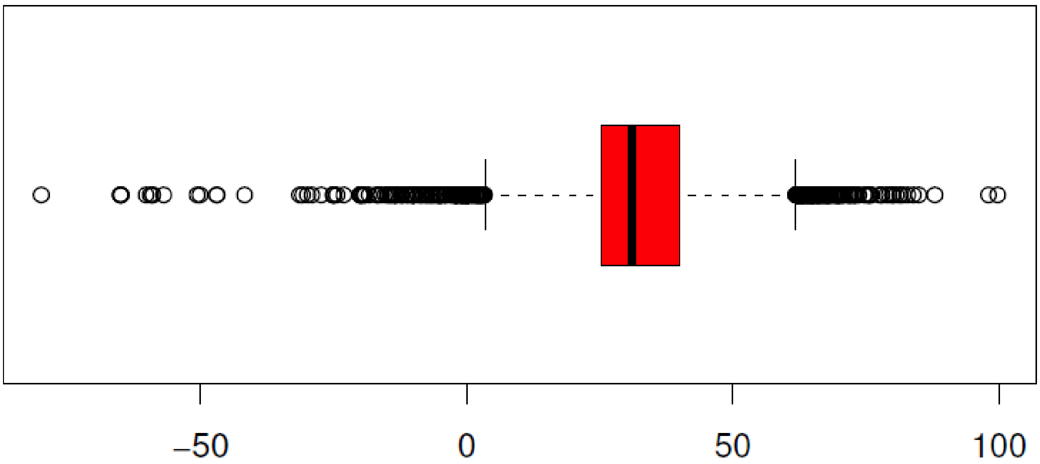


Figure 17. Box-plot of price

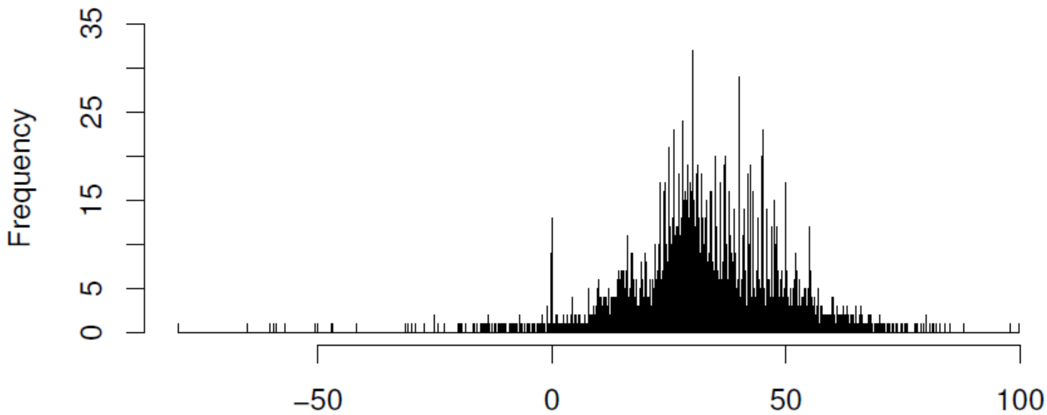


Figure 18. Full price histogram

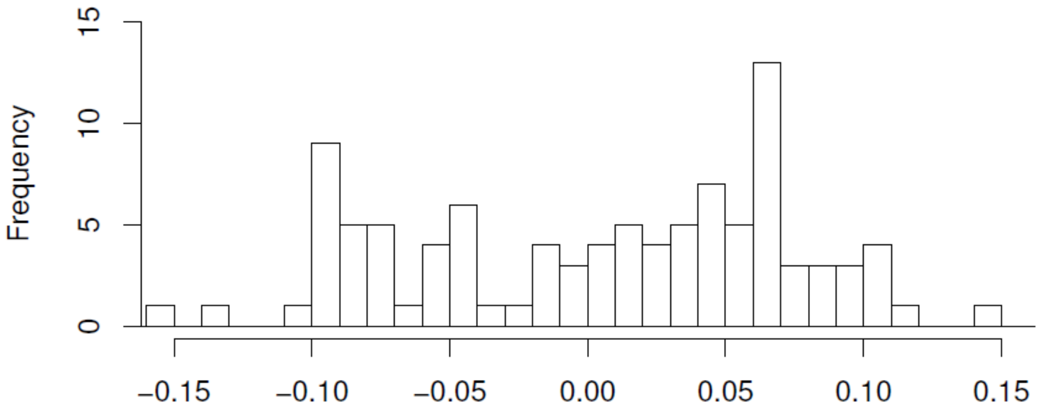


Figure 19. Price histogram around 0€/MWh

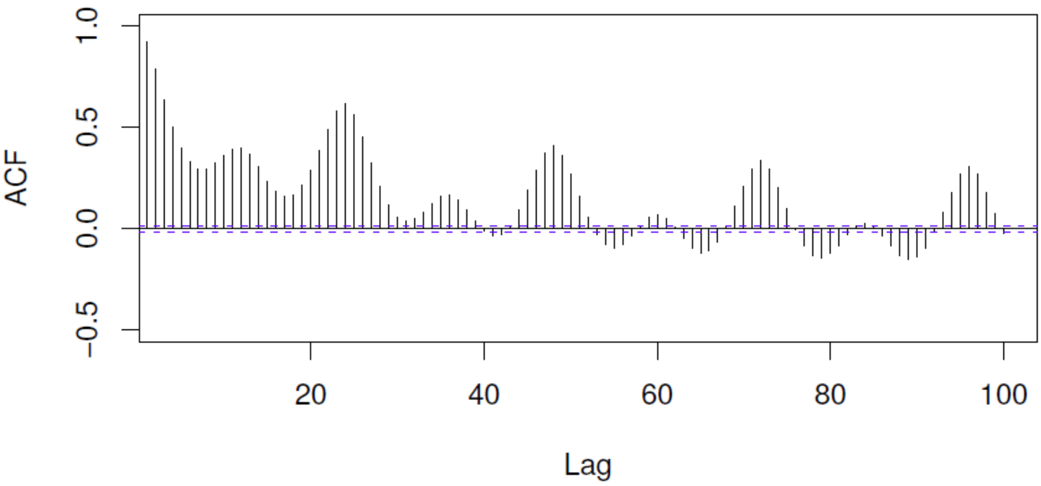


Figure 20. Price autocorrelation function

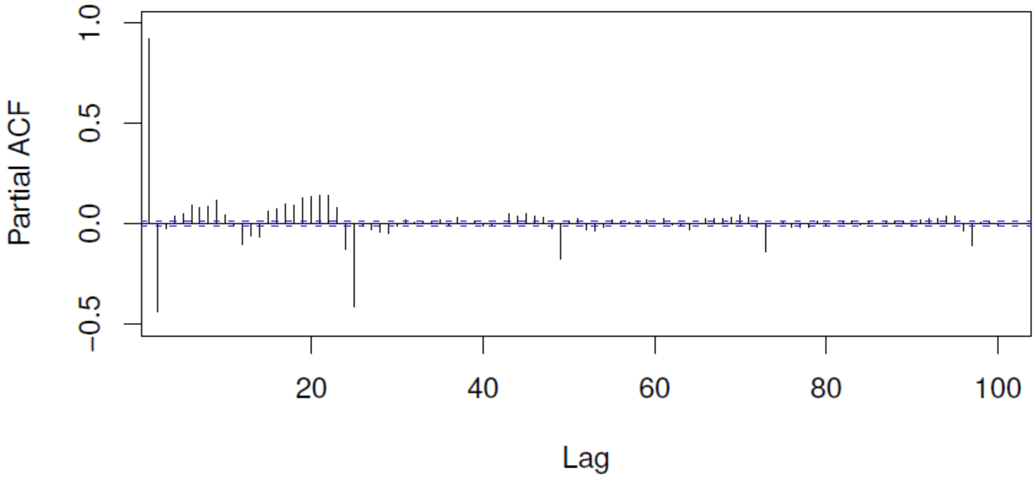


Figure 21. Price partial autocorrelation function

Null Hypothesis: PRICE has a unit root
Exogenous: Constant
Lag Length: 28 (Automatic - based on SIC, maxlag=43)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−17.13532	0.0000
Test critical values:	1% level	−3.430552	
	5% level	−2.861513	
	10% level	−2.566797	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: PRICE has a unit root
Exogenous: Constant
Bandwidth: 19 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		−24.45028	0.0000
Test critical values:	1% level	−3.430551	
	5% level	−2.861513	
	10% level	−2.566797	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: PRICE is stationary
Exogenous: Constant, Linear Trend
Bandwidth: 86 (Newey-West automatic) using Bartlett kernel

		LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic		0.109461
Asymptotic critical values*:	1% level	0.216000
	5% level	0.146000
	10% level	0.119000

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Table 6. Unit root and stationarity tests for the price time series

B. Periodograms

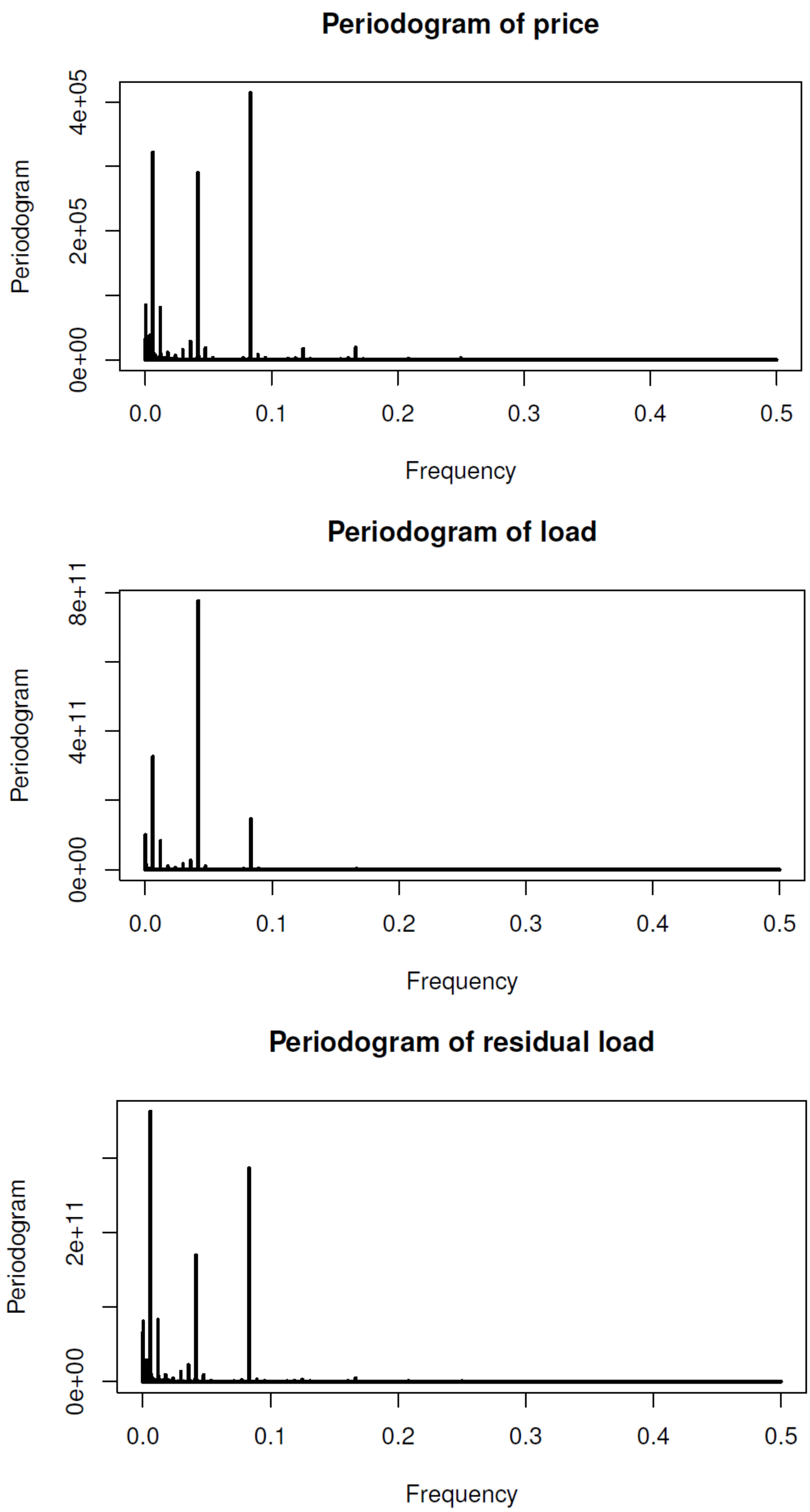


Figure 22. Periodograms of price, load, and residual load

C. Mathematical expressions of the marginal effects

C.1. Marginal effects on price

We show here how to derive the expression for the marginal effects in each regime. For simplicity of presentation, we use the centred variable $X = Price - \langle Price \rangle$ (where $\langle \cdot \rangle$ is the temporal mean operator), we denote by $W = Wind/Load$ and $S = Solar/Load$ the relative shares of wind and solar productions, and we drop the time index. W we have the following computations:

$$
\begin{aligned}
& \sinh^{-1} X = \beta_0 + \beta_1 W + \beta_2 S + \beta_3 RLoad + \varepsilon \\
\Rightarrow \quad & \frac{\mathrm{d}Price}{\sqrt{X^2+1}} = \beta_1 \mathrm{d}W + \beta_2 \mathrm{d}S + \beta_3 \mathrm{d}RLoad \\
& \qquad = \beta_1 \left(\frac{\mathrm{d}Wind}{Load} - \frac{Wind}{Load}\frac{\mathrm{d}Load}{Load} \right) + \beta_2 \left(\frac{\mathrm{d}Solar}{Load} - \frac{Solar}{Load}\frac{\mathrm{d}Load}{Load} \right) \\
& \qquad + \beta_3 \left(\mathrm{d}Load - \mathrm{d}Wind - \mathrm{d}Solar \right)
\end{aligned}
$$

If we suppose that solar and wind productions have no influence on each other nor on the load (and reciprocally)[5], we obtain the following expressions for the marginal effects (partial derivatives), for absolute variations of load and wind and solar productions:

$$\frac{\partial Price}{\partial Wind} = \sqrt{X^2+1} \left(\frac{\beta_1}{Load} - \beta_3 \right) \tag{8}$$

$$\frac{\partial Price}{\partial Solar} = \sqrt{X^2+1} \left(\frac{\beta_2}{Load} - \beta_3 \right) \tag{9}$$

$$\frac{\partial Price}{\partial Load} = \sqrt{X^2+1} \left(\beta_3 - \beta_1 \frac{Wind}{Load^2} - \beta_2 \frac{Solar}{Load^2} \right) \tag{10}$$

Finally, we compute these marginal effects for each time step and take their mean and standard deviation, as shown in table 5[6].

C.2. Marginal effects on expected duration

As stated in section 4, the marginal effects computed below are not mathematically rigorous, since we use formulas for expected duration and stationary distribution that are valid only for homogeneous Markov chains, but we hope that it will nevertheless give an idea of the magnitude of the marginal effects. First, let us recall the aforementioned formulas:

$$\mathbb{E}\tau_i = \frac{1}{p_{ij}} = \frac{1}{1-p_{ii}} \quad \text{and} \quad \pi_1 = 1 - \pi_2 = \frac{p_{21}}{p_{12}+p_{21}}$$

We inject the expression of the time-varying probabilities in the equation giving the expected durations using W and S as before for the sake of simplicity:

[5] The opposite could be possible, for instance, if there was excess RES production so that wind or solar production must be shedded.

[6] The standard deviations are computed directly from the time series generated by equations 8-10. To obtain robust standard deviations, one should use bootstrap methods in order to take into account the variance of the estimators.

$$p_{i1} = \frac{1}{1+\exp\left(-\alpha_0^{(i)} - \alpha_1^{(i)} W - \alpha_2^{(i)} S\right)} \quad \text{and} \quad p_{i2} = \frac{1}{1+\exp\left(\alpha_0^{(i)} + \alpha_1^{(i)} W + \alpha_2^{(i)} S\right)}$$

so that we have for the expected durations:

$$\begin{aligned}
& \mathbb{E}\tau_1 = 1+\exp\left(\alpha_0^{(1)} + \alpha_1^{(1)} W + \alpha_2^{(1)} S\right) \\
\Rightarrow \quad & \mathrm{d}\mathbb{E}\tau_1 = \alpha_1^{(1)} \exp\left(\alpha_0^{(1)} + \alpha_1^{(1)} W + \alpha_2^{(1)} S\right) \mathrm{d}W + \alpha_2^{(1)} \exp\left(\alpha_0^{(1)} + \alpha_1^{(1)} W + \alpha_2^{(1)} S\right) \mathrm{d}S \\
\Rightarrow \quad & \frac{\mathrm{d}\left(\mathbb{E}\tau_1 - 1\right)}{\mathbb{E}\tau_1 - 1} = \alpha_1^{(1)} \mathrm{d}W + \alpha_2^{(1)} \mathrm{d}S \\
& \qquad = \alpha_1^{(1)} \left(\frac{\mathrm{d}Wind}{Load} - \frac{Wind}{Load}\frac{\mathrm{d}Load}{Load}\right) + \alpha_2^{(1)} \left(\frac{\mathrm{d}Solar}{Load} - \frac{Solar}{Load}\frac{\mathrm{d}Load}{Load}\right)
\end{aligned}$$

and

$$\begin{aligned}
& \mathbb{E}\tau_2 = 1+\exp\left(-\alpha_0^{(2)} - \alpha_1^{(2)} W - \alpha_2^{(2)} S\right) \\
\Rightarrow \quad & \mathrm{d}\mathbb{E}\tau_2 = -\alpha_1^{(2)} \exp\left(-\alpha_0^{(2)} - \alpha_1^{(2)} W - \alpha_2^{(2)} S\right) \mathrm{d}W - \alpha_2^{(2)} \exp\left(-\alpha_0^{(1)} - \alpha_1^{(2)} W - \alpha_2^{(2)} S\right) \mathrm{d}S \\
\Rightarrow \quad & \frac{\mathrm{d}\left(\mathbb{E}\tau_2 - 1\right)}{\mathbb{E}\tau_2 - 1} = -\alpha_1^{(2)} \mathrm{d}W - \alpha_2^{(2)} \mathrm{d}S \\
& \qquad = -\alpha_1^{(2)} \left(\frac{\mathrm{d}Wind}{Load} - \frac{Wind}{Load}\frac{\mathrm{d}Load}{Load}\right) - \alpha_2^{(2)} \left(\frac{\mathrm{d}Solar}{Load} - \frac{Solar}{Load}\frac{\mathrm{d}Load}{Load}\right)
\end{aligned}$$

Finally, assuming again that solar and wind productions have no influence on each other nor on the load, we can isolate the marginal effects and take their mean, so that:

$$\frac{\frac{\partial\left(\mathbb{E}\tau_1 - 1\right)}{\mathbb{E}\tau_1 - 1}}{\partial Wind} = \frac{\alpha_1^{(1)}}{\langle Load \rangle} \; ; \; \frac{\frac{\partial\left(\mathbb{E}\tau_1 - 1\right)}{\mathbb{E}\tau_1 - 1}}{\partial Solar} = \frac{\alpha_2^{(1)}}{\langle Load \rangle}$$

$$\frac{\frac{\partial\left(\mathbb{E}\tau_2 - 1\right)}{\mathbb{E}\tau_2 - 1}}{\partial Wind} = -\frac{\alpha_1^{(2)}}{\langle Load \rangle} \; ; \; \frac{\frac{\partial\left(\mathbb{E}\tau_2 - 1\right)}{\mathbb{E}\tau_2 - 1}}{\partial Solar} = \frac{\alpha_2^{(2)}}{\langle Load \rangle}$$

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