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DESIGNING EFFICIENT CAPACITY MECHANISMS: BIDDING BEHAVIOR AND PRODUCT DEFINITION

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DESIGNING EFFICIENT CAPACITY MECHANISMS: BIDDING BEHAVIOR AND PRODUCT DEFINITION

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Abstract

In many countries, capacity markets have been put in place to supplement wholesale markets revenues to ensure an adequate generation capacity to maintain security of supply. This paper studies the bidding behavior in those markets and how it can be affected by different capacity product designs. A capacity market allows producers to lock in revenues in advance in exchange for their commitment to being available over a future period on wholesale markets. Producers' participation depends on the opportunity cost of making the investment available. When the commitment is made, the profitability of the plant is uncertain. The canonical framework is based on a net present value model, where the capacity bid is equal to the expected loss on the energy market. However, this does not recognize managerial flexibility and assumes that the plant cannot react to future market conditions. Thus, we propose a novel approach to conceptualize capacity bids using real options theory, where the opportunity cost is represented as an option on the spread that drives the profitability of the plant. First, we define a bid in a one-period capacity market as a European Put Option. Then, we expand to a multi-period setting in which capacity bids can be evaluated as a modified Basket Option. Our model provides new insights on the interplay between the product/commitment duration and on capacity bid. Using the real options approach, the model presents a first attempt to untangle the different drivers of the opportunity cost for providing capacity availability. We analyze the determinants of the option value concomitantly with the length of the procurement and deduce some policy implications for the product's design. Finally, we provide a numerical illustration of this issue using data from the French power system.

Keywords: Market Design, Real Options, Capacity Markets, Electricity Markets.

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I. INTRODUCTION

In current power systems, producers do not always receive enough revenue to cover their production costs even though they are deemed necessary to reach the first best investment mix. Electricity prices can be constrained due to political reasons with price caps [24] or can send distorted price signals due to technical and noneconomic interventions on the market [20]. Other reasons can be found in that electricity prices do not consider the correct value of an additional capacity, for instance, due to the public good nature of capacities during high demand periods [19] or because some externalities are not correctly internalized [22]. At the same time, the risk of not having enough investment poses a significant threat. Indeed, the absence of adequacy between the capacity installed and the electricity demand, combined with the difficulty of implementing efficient rationing, leads to high system costs. It has been illustrated by the rolling blackouts in the Texas system last winter or during hot summers in California.

One solution to restore the right level of investment could be the implementation of capacity remuneration mechanisms. They provide the producers with an additional remuneration stream to increase and maintain the optimal level of investment. There are currently various implementations ranging from capacity payments paid directly to the producers to more complex designs with actual markets, where the price emerging from the confrontation between a supply and a demand for capacity makes the additional remuneration. They are usually denominated as capacity markets. Each participating producer makes a price-quantity offer for a capacity on the supply side of those competition-based mechanisms. If a producer sells a capacity, he receives an additional price, and it legally forces the investment to be available over a specific period in the future.

In this paper, we mainly investigate two research questions (i) how to model capacity bids in the context of uncertainty and managerial flexibility to operate or close the plant (ii); how bids depend on multiple key design features. We tackle those issues by stating that participation in a capacity market implies a specific opportunity cost for the bidder, which is the fundamental driver for its bidding behavior. To do so, we analyze the opportunity cost determinants associated with the decision to be available, allowing a more detailed comparison with the marginal value of an available capacity, independent of the product design. Therefore, the subsequent analysis sheds light on effectively setting up a mechanism based on competition, where the price signal improves economic efficiency. In this paper, the capacity price encourages producers to invest and stay open when wholesale markets cannot send the proper price signal. Therefore, any deviation of the price from the actual value of an additional capacity for the system can cause an adverse effect. We stress that both market design theory and practitioners must consider the practical limits imposed by the actors' behavior in the face of specific rules.

To our best knowledge, we are the first to use a methodology other than the net present value framework to analyze the bidding behavior in a perfectly competitive capacity market. Namely, we state that the opportunity cost of participating in a capacity market is equal to the option value of the availability decision. Such conceptualization sheds new light on how prices emerge in a capacity market. There also has been no formal analysis of the link between the bids in a capacity market and the duration of a capacity product. Therefore, our paper deepens policy perspectives for the practical implementation of capacity markets.

The opportunity cost of participating in the capacity market is well known in the literature, and some papers have highlighted the need to grasp the role of product design better when assessing the efficiency of those mechanisms. They have underlined the necessity of understanding the opportunity cost drivers when selling an availability to refine the study of capacity prices and help choose the right product design. In this paper, we underline the multidimensional aspect of this issue with two rationales: (i) the interdependence between the wholesale market and the capacity market (ii) the managerial flexibility the investment encompasses.

We start our model by recalling the fundamentals behind a single power system investment decision from a private producer perspective. Then we introduce a simplified capacity market where the representative producer can bid in an auction mechanism a capacity product that forces the investment to be open during a specific period determined before the auction is set.

First, we use a net present approach where the producer only offers the expected opportunity cost associated with the capacity product. In this case, he bids the expected revenues over the procurement duration net of the fixed cost associated with the decision to stay available. We show that a longer product always implies a lower or equal bid than the sum of expected bids for shorter products. In both cases, the bids are always equal to the expected loss. Otherwise, the producer makes a null bid. This first approach implies for the producer a comparison between only two alternatives (i) being available during the whole procurement period or (ii) closing during the same periods.

Our main contribution lies in studying the bid as an option value associated with the possibility to close temporarily but irreversibly to avoid fixed costs. First, we use the standard option pricing theory to value a simplified version of a capacity market where the period during which is the plant has to be available, called the transaction phase, covers only a single wholesale market clearing. Under this case, the capacity product is equivalent to a European Put Option where the exercising date is the transaction phase, the underlying being the wholesale profit, and the strike price is the fixed cost associated with the decision to stay open. Under the real options framework, the bid on the capacity market is strictly equal to the option value. Then, we expand this analysis to a multiperiod transaction phase, and we treat the capacity product as a form of Basket Option where the asset price portfolio is the expected revenue generated over the procurement period. It allows us to compare this option value with the sum of the option value for shorter products. Using the real options framework to assess the bidding behavior in a capacity market, we find that it significantly differs from the net present value framework. First, bids are always higher under the real options framework, meaning that producers place a positive value on the possibility to close to avoid some costs. Second, the drivers behind the bids have different effects on their value compared to the net present value framework. We provide comparative statistics on the bids value and the difference between the two frameworks. We find that the length of the transaction phase constantly increases the bid when using the real options theory while having an ambiguous effect on the net present value bid. The volatility on the wholesale market and the policy instrument, the waiting time between the sale of the capacity product, are also analyzed. They both have ambiguous effects on the capacity bids depending on a set of conditions on the bid drivers. Finally, we find the reverse effect for the product design dimension with a higher bid with a longer transaction phase than the sum of expected bids with shorter products.

We test our results by calibrating the model to the French electricity system. We use realized data for a CCGT (gas) power plant to simulate a bid in the capacity market and compare the outcomes with realized prices observed on the French capacity market. While the results are highly sensitives to the assumptions regarding the drivers of the bids, we find that the real options framework can explain auction outcomes. Our model also stresses that a change of volatility for the investment revenue, due, for instance, to the increasing share of renewables, can significantly affect the bidding behavior in a capacity market. Similarly, choosing the duration between the auction date and the transaction phase when designing the capacity market has important implications when looking for the least cost design.

Using both the theoretical framework and the numerical illustration, we provide a policy discussion for the design of capacity markets. Namely, using a real options framework sheds light on the role of penalty in the capacity market imposed onto producers who choose not to be available during the procurement duration despite having sold a capacity product. Similarly, the cost associated with the decision to close can also be included in the analysis. Another crucial point can be made regarding the difference between existing and new capacity. While for the former, the opportunity cost of being available is only made concerning a single capacity product, for the latter, the decision to enter is more complex. Indeed, it is based on the expected revenue made during the investment lifetime, including future capacity prices. In this case, the effect of different procurement duration can be significant.

II. LITERATURE REVIEW

In terms of capacity markets, a vast literature has studied their effect on investment decisions. Such assessment has been realized in simplified models such as classical Nash equilibrium models [14], with sometimes a representation of strategic actors [28], and stochastic optimization models where the market is mimicked using a minimization cost function [13]. Other models tried to replicate the complex environment in which those mechanisms have been implemented by representing different flux between agents and their decisions' implications. System Dynamics models studies dynamically the effect of capacity markets on investment decisions [10], while Agent base Models use a bottom-up approach to analyses the interactions of specific agents in the power system [5]. In most papers, regardless of the types of models, they find that capacity markets significantly improve energy markets' efficiency by increasing investment value and reducing the capacity adequacy issue.

Taking a different angle, we base our work on *single project valuation models* which are less used in this context. The advantage of this type of model relies on the possibility of finely representing the components of investment value for a producer, their evolution, and technological constraints. More specifically, it allows for a hypothetical investment to represent both its future revenues and the impact of the additional remuneration on its value dynamically. On the other hand, our model lacks a system view, with no representation of market feedback, technology competition, and market power. Most of the single project valuation model stream applied to power investment focused on real options analysis to study the different value of potential managerial decisions, such as investment in renewable energy under price uncertainty ([15], [17]), conventional investments under policy and finance uncertainty ([23]) or the effect of different support mechanisms for renewable on investment decision ([17]). To our knowledge, only one paper has taken the single project valuation to capacity markets: [18] found that exogenous capacity payments significantly modify the value of new gas power plants, especially when the quantity of renewable is high. Therefore, we expand this approach by endogenizing the payments while using actual data to deduce the investment value and studying different product designs.

The fundamental driver behind bid formation in capacity markets, developed for instance by [31], is that participation in such a market creates an obligation to be available in a future period on the energy market. Therefore, selling a capacity generates an indirect cost, which could be described as an opportunity cost. The opportunity cost of participating in a mechanism is the cost of being available during a predefined future period, which would not have been incurred if the investment was not producing during the same period. Failures and constraints can lead to insufficiently high prices to cover their costs, even though they are necessary for the system. That is when the marginal production cost is lower than some willingness-to-pay of unserved consumers. Consequently, forcing an actor to produce when it is potentially at a loss entails a positive opportunity cost but allows the energy to be efficiently dispatched.

Some papers seeking to reproduce the interdependence of the actors and the different production decisions in power markets are based on this principle [2, 5, 30]. [12] shows, for instance, how a monopoly offers on a capacity market when the latter has to give up exporting profit in a foreign market whose price is higher than the price on the national market due to the obligation to be available. The offer on the capacity market is made at a price equivalent to the loss of opportunity to make a profit on the foreign market. [8] proposes the term of *allocation externality* to characterized the link between capacity bids and energy profits. In his setup, incumbents are dumping capacity prices to avoid new entry into energy markets. Because new entry is made possible with the capacity market, the energy profit could be lower due to higher competition. Therefore, it can be strategic to make losses on capacity markets to prevent more significant losses in the energy market. The few papers modeling the reliability option markets are also enlightening about this approach.[1] During periods of scarcity of demand, that is, when the plant is needed, producers undertake to pay back on demand the difference between the energy price received on the energy markets and the strike price of the

[1] These mechanisms, close to capacity markets, are based on the exchange of financial options between the actors holding the investment and demand. Initially held by the players, these options are sold on a market, which constitutes remuneration for their capacity.

obligation [11]. In those models, the opportunity costs, and thus bids for these options, are equal to the amount transferred on demand [26, 29].

In a similar approach to this paper, [4] uses the real options theory to analyze the bidding behavior in a Reliability Option mechanism. They describe multiple complex frameworks to derive the opportunity cost of participating in those mechanisms. However, the fundamentals for the bids on those mechanisms are different from our set up[2] and they do not address the product design dimension. Finally, our work is close to the paper of [27]. They provide new insights on bidding behavior for renewable auctions by also using real options. However, they again study a different framework from ours[3]

In this work, simulations in the single project model allow defining opportunity costs associated with participation in a capacity market. We represent the possibility that the investment will not recover enough of its costs when forced to produce, which creates the implicit cost associated with the decision to close. We show that its value is significantly impacted by the profit drivers forecast and the capacity product's design. Current debates on the design of capacity markets have not yet determined the optimal capacity product if it exists. There is a coexistence of these products in most current markets, which underlines the importance of modeling their potential effects on investment values. Table 1 recapitulates the different variations and illustrates some example of current capacity markets and their relative product design. Note that some markets also include a distinction between new capacity and existing capacity. The former can either buy the long or the short product, while the latter is usually only allowed to buy the shorter product.

Transaction phase			
Monthly	Quaterly	Yearly	Multiyear
-	-	France	France
CAISO	-	CAISO	
-	SPP	-	-
-	-	PJM	-
-	-	UK	UK
-	-	Poland	Poland
-	-	Belgium	-
-	-	ISO NE	-
NYSO	-	NYSO	-
-	-	-	Ireland
-	-	-	Italy
-	-	Greece	-

Table 1: Product designs used in the model and actual implementations

[2]For instance, the strike price is explicit in reliability options mechanisms and the comparison of different transaction phases do not entail the same implications.

[3]Producers bid for the price they will receive once the investment is made, without knowing their production costs. Moreover, the option is covering a single period.

The impact of the length of the contractual period is even less discussed formally in the literature. [9] emphasizes the importance of carrying out such analysis to improve the understanding of a capacity market. To our knowledge, [6] and [7] are the only ones to have addressed this issue qualitatively and quantitatively. In [6] they underline the unsettled tradeoff between the financing costs and complexity costs of shorter products, and the costs of capacity over procurement and costs of excluding flexible generators of longer products. In [7], they investigate the implications of the length of capacity products procured when there is seasonal variation in both the electricity load and the electricity generation. Using a Nash-equilibrium approach with investments and bidding behavior, they illustrate the efficiency tradeoffs associated with introducing multiple shorter capacity products instead of procuring a single annual capacity product and derive the optimal length of a capacity product. Our model has the same spirit, but leaving aside the market representation, we focus on the coexistence between engaging in the capacity market and on managerial option (closing) and adapting our model with more detailed technological characteristics.

Similarly, [3] build a complex model using the System Dynamic approach to understand how closing can modify the implementation effect of a capacity market regarding an initial sub-optimal energy-only market. However, contrary to our model, the author does not expand his analysis on multiple product designs. We also allow a high degree of flexibility in the mothballing decision by representing different closing periods.

III. MODEL ASSUMPTIONS

3.1. Investment and wholesale market

We focus on a hypothetical setup with a single risk-neutral producer. He can invest in a unique power plant of a specific technology used to sell electricity at a future price on the wholesale market at date t and with a price p_t. If the producer enters the market by building his investment, he sustains an initial investment cost of c^I. Every $\bar{n}^{om}$ dates he can choose to stay open during a following period of length n^{om}, called the closing period. If the producer decides to stay open, he sustains a fixed cost of c^{om} called the periodic fixed cost. Those costs typically include operation and maintenance costs, leases, or wages. He can also produce whenever the wholesale price is above the marginal production cost c^v and sell its electricity on the wholesale market. The variable costs usually include fuel cost and carbon cost. Otherwise, if the investor chooses to close temporarily, he avoids the fixed cost but cannot produce. We normalize the capacity level, so one unit of capacity produces one unit of electricity. It is similar to assume an absence of economies of scale, where producers with discrete capacity value would make piece-wise bids.

We define the inframarginal rent collected at a date t as the net wholesale revenue as $\pi_t = (p_t - c^v)^+$. We assume it is uncertain for the investor at any date prior to t. We model this uncertainty using a stochastic process $(\pi_t)_{t \geq 0}$. This stochastic process follows a Geometric Brownian Motion such as it satisfies the stochastic differential equation [4]:

$$\Delta\pi = \mu\pi_t dt + \sigma\pi_t \Delta Z_t \quad (1)$$

With μ and σ respectively, the drift and the volatility of the Brownian Motion and ΔZ_t are the increments of a standard Brownian motion. This assumption regarding the uncertainty of the profit drivers is commonplace in commodity markets, especially when studying investment decisions in the electricity sector (see, for instance, [23], and [27]). It allows capturing the randomness of the future variable cost, which follows the price of other commodities such as oil and gas, and the intrinsic uncertainty of electricity prices which depends, for instance, on weather conditions, demand patterns, and carbon prices.

We consider a risk-neutral investor, so we define a constant risk-free interest rate r which is also used as an discount rate in our model. Therefore, the rent process is defined by:

$$\Delta\pi^* = r\pi_t^* dt + \sigma\pi_t^* \Delta Z_t^* \quad (2)$$

We follow the canonical notation where ΔZ_t^* is the increments of the Brownian motion under the equivalent martingale measure $\mathbb{Q}$. We also want to study the sum of inframarginal rents' distribution; we make the following assumption.

Assumption 1. *The sum of inframarginal rent collected by a producer is log-normally distributed.*

More precisely, if $\int_{i=0}^{n} \pi_t$ represents the sum of the stochastic process values over a period n with μ and σ respectively, the drift and the volatility, then the sum follows a log-normal distribution. Alternatively, this is similar to say that if $\int_0^n \pi_t = e^X$ then:

$$X \sim \mathcal{N}(m, v) \quad (3)$$

with $m = 2\ln[M] - 0.5\ln[V^2]$ and $v^2 = \ln[V^2] - 2\ln[M]$ such that M is the expected value and V^2 the second-order moment of the sum. We use this assumption as there is no explicit analytic expression of the distribution of the sum of Geometric Brownian Motion. This analytic approximation is commonplace in finance theory and relies on approximating the unknown distribution by another tractable one (see, for instance, [25] and [21]). More specifically, we use a moment matching method where the moments of the sum distribution are matched with the moment of the log-normal distribution. In the appendix, we provide more details for these assumptions when demonstrating the results of proposition 4. Finally, we make the following assumption regarding the relationship between the different closing periods.

Assumption 2. *A closing decision for a specific period does not affect the profit or the producers' cost for other periods.*

For instance, closing the investment does not raise the production cost over the following periods nor decreases the revenue perceived on the wholesale market.

3.2. Producer behavior and market efficiency

Following the canonical theory for investment decisions in the electricity sector, the sum of inframarginal rents collected during the entire lifetime T of the power plant should be covering the initial investment cost incurred at $t = 0$. However, when we take into account the periodic fixed cost, the producer enters the market only if the following equality holds:

$$\int_0^T e^{-rt}\mathbb{E}_0^*\left[\pi_t\right]dt = c^I + c^{om}\sum_{i=0}^{\bar{n}^{om}-1}\int_0^{n^{om}} e^{-r(t+i\times n^{om})}dt \tag{4}$$

With $\mathbb{E}_0^*$ the expectation operator at date $t = 0$ with respect to the equivalent martingale measure $\mathbb{Q}$, T the power plant's lifetime, $\bar{n}^{om}$ the number of time the producer has to choose to stay open and to pay the periodic fixed cost. The first left term represents the sum of net expected revenue made on the wholesale market, the first right term is the investment cost, and the last right term represents the actualized sum of the periodic fixed cost. For tractability, we made c^{om} periodic occurred at each period t, but recall that it is always sustained whenever the investment stays opened during a period of n^{om}. The equality in equation 4 is similar to stating that the investment NPV is at least null. The investment NPV at time $t = 0$ can be defined as follow:

$$\Pi_0 = \int_0^T e^{-rt}\mathbb{E}_0^*\left[\pi_t\right]dt - c^I - c^{om}\sum_{i=0}^{\bar{n}^{om}-1}\int_0^{n^{om}} e^{-r(t+i\times n^{om})}dt \tag{5}$$

Following a similar approach, the power plant chooses to be available whenever it is profitable. The condition for an opening for each period n^{om} is given by the following equality:

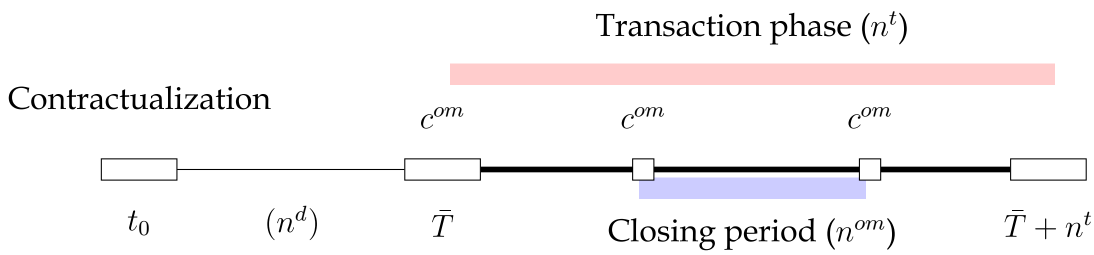
$$\int_0^{n^{om}} e^{-rt}\mathbb{E}_0^*\left[\pi_t\right]dt = \int_0^{n^{om}} e^{-rt}c^{om}dt \tag{6}$$

For tractability, we assume that the decision is made at the beginning of a closing period.

While the canonical theory states that a producer should be producing, and therefore be available, each time the electricity price is above its marginal cost, the introduction of period fixed costs can induce a risk of inefficiency on the power system. Namely, assuming that the power plant is indeed necessary for the system, as soon as the power plant closes because the inframarginal period collected on a specific period is below the corresponding fixed cost, it generates a net welfare loss for the system [4]. Given the previous assumptions on the investment, one can quickly compute the optimal number of periods t over the lifetime during which $\pi_t \geq 0$ and compare it to the number of periods over the lifetime during which the investment is open and $\pi_t \geq 0$. A difference between those results would show that the power plant inefficiently behaves. Such inefficiency could be due to a price cap on the energy market, the effect of the non-economic intervention of the system operator, or unpriced externalities.

3.3. The capacity market

If a regulator decides that this power plant is necessary for the system, she implements a capacity market to encourage the producer to invest and be available. To so, she defines a capacity product with a specific duration called the *transaction phase* and notes n^t. She organizes the transaction of this product via a market mechanism such as an auction at a contractualization date noted t_0. Once the producer has sold the product, he is legally bound to be available during the transaction phase, that is, to be on the market during a transaction phase of length n^t. This period starts at a predefined date noted $\bar{T}$, with n^d the distance between the auction date and the starting date of the transaction phase. The regulator can use multiple instruments to check for availability, such as unannounced tests or verifying book orders on the energy market. One can note that the transaction phase is not necessarily equal to the closing period, which is investment-specific. It can either be lower, equal, or superior. The following figure illustrates the design of a capacity market where the transaction phase implies three closing decisions for the investment.



For the producer, the capacity price received in the capacity market enters its profit as a second stream of remuneration in addition to the revenue made on the wholesale market. Similarly to the closing periods, we define $\bar{n}^t$ the number of times a capacity auction is set during the investment lifetime. Using the NPV of the investment over its lifetime, the final NPV with the capacity market is equal to:

$$\Pi_0^{cm} = \int_0^T e^{-rt}\mathbb{E}_0^* [\pi_t]\, dt - c^I - c^{om} \sum_{i=0}^{\bar{n}^{om}-1} \int_0^{n^{om}} e^{-r(t+i\times n^{om})} dt + \sum_{i=0}^{\bar{n}^t} e^{-r(i\times n^t)} p_i^c \qquad (7)$$

[4]Our approach to the market efficiency implies that while the wholesale price is below an optimal value that covers the fixed cost, it optimally sends short term signals. To say it differently, if the wholesale price would have been optimal, then occurrences of prices below the marginal cost are the same as the occurrences with the inefficient price. Our analysis could be extended to the case where prices are also inefficient concerning the marginal cost, but it implies additional assumptions to differentiate between periods when both optimal and inefficient prices are above the marginal cost and when they are not.

With p^c the capacity price which is received at every capacity auction. By construction, auctions are set up at an interval of n^t. As we do not model the competitive process in the paper, we simplify the analysis by assuming that the bids the investor makes on the capacity market are equal to the price he receives. Such assumption holds under the case the investment is always the marginal bidder in a uniform auction, or if he bids truthfully in a pay-as-bid type auction or in a bilateral marketplace[5]. We restrict the most straightforward design in the following sections and provide more extensions in the policy discussion section.

IV. THE NET PRESENT VALUE FRAMEWORK

We start our analysis by describing the bidding behavior of a producer who offers only its net present opportunity cost on the capacity market associated with an existing investment. It allows to precise the definition and the bid's rationals in a capacity market. We discuss the relation between the product design and the expected inframarginal rent net of the periodic fixed cost, which is a basis for the canonical approach to model bids in the capacity market. Those results also serve as a reference value to compare the bidding behavior when the option value is taken into account using a real options framework.

4.1. The inframarginal rent and the opportunity cost

The opportunity cost associated with participation in a capacity market is based on the dichotomy between sunk and non-sunk fixed costs incurred when the producer decides to produce. It is crucial as some fixed costs could be considered sunk before participating in a capacity market. Indeed, recall that fixed costs are decomposed into two parts: (i) Investment costs, which incur at the power plant's first activation. (ii) periodic fixed costs, which incur periodically and irrevocably. When considering entering the market, investment and periodic fixed costs are still pending and avoidable. Consequently, the time horizon used to compute the opportunity cost of entering the market should be based on the entire project lifetime. Indeed, when the producer compares the decision to enter the market at date $t = 0$, he faces a tradeoff between (i) receiving the asset value; and (ii) never entering the market, which translates into a null value. On the other hand, if the producer has already invested, periodic fixed costs are the only fixed costs avoidable, and the time horizon is limited to the closing period. Therefore, when the producer forecasts the decision to participate in the capacity market at date t_0, he faces a tradeoff between having to open the power plant and potentially incurring net losses; or leaving temporary the market at no cost[6].

The cost associated with such an opportunity over a transaction phase is the difference between the sum of the periodic costs linked to the decision to stay open during the obligation to produce and the profits made only during the period covered by the capacity product. Formally we note B_0 the initial bid made before the investment is made, and b_{t_0} the bids made at every auction date t_0 during the investment lifetime. For instance,

[5]See for instance [27] for a discussion on the truthful behavior in competition based mechanisms for investments in electricity production.

[6]We assume in the model extensions a cost associated with the possibility to close temporarily.

assume a product with a transaction phase of n^t sold at t_0 with a periodic fixed cost sustained over an identical period ($n^{om} = n^t$), if we assume that the producer offers its net present opportunity cost on the capacity market, then the following equality based on the condition 6 must hold:

$$\int_0^{n^t} e^{-rt}\mathbb{E}_0^*\left[\pi_t\right] dt + p^c = \int_0^{n^t} e^{-rt} c^{om} dt \tag{8}$$

Having assumed that producers truthfully bid then the bid value for an existing plant on a capacity market is:

$$b_{t_0} = e^{-rn^d}\left[\int_0^{n^t} e^{-rt} c^{om} dt - \int_0^{n^t} e^{-rt}\mathbb{E}_0^*\left[\pi_t\right] dt\right]^+$$

Under the net present value framework, the opportunity cost of participating in a capacity market is equal to the expected short-term *Missing Money*, that is, the expected loss of staying available due to the existence of fixed periodic costs. Given the bids for an existing power plant, we can now define the bid for a new investment. It is based on the wholesale revenue but also on the expected bids on the capacity market. The following equality based on the condition 4 must hold and implies that the investment NPV given the capacity prices is null:

$$\int_0^{T} e^{-rt}\mathbb{E}_0^*\left[\pi_t\right] dt + \sum_{i=0}^{\bar{n}^t-1} e^{-r(i\times n^t)} b_{i\times n^t} + p^c = c^I + c^{om}\sum_{i=0}^{\bar{n}^{om}-1}\int_0^{n^{om}} e^{-r(t+i\times n^{om})} \tag{9}$$

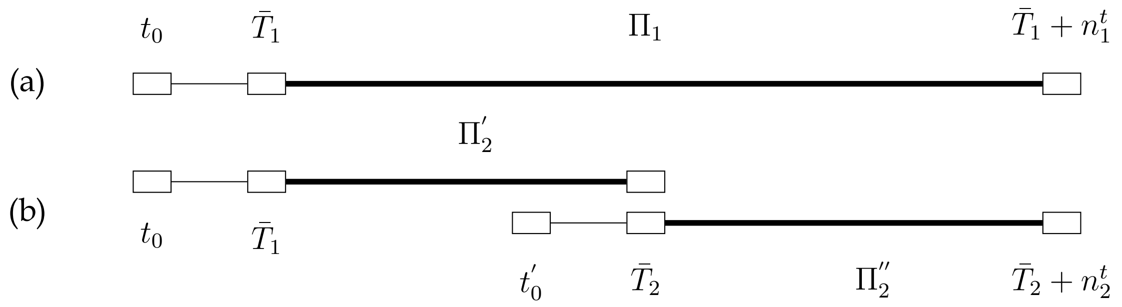
Having assumed that producers truthfully bid, then the bid value for an existing plant on a capacity market is:

$$B_0 = \left[c^I + c^{om}\sum_{i=0}^{\bar{n}^{om}-1}\int_0^{n^{om}} e^{-r(t+i\times n^{om})} - \int_0^{T} e^{-rt}\mathbb{E}_0^*\left[\pi_t\right] dt - \sum_{i=0}^{\bar{n}^t-1} e^{-r(i\times n^t)} b_{i\times n^t}\right]^+ \tag{10}$$

With $\bar{n^t}$, the number of times a capacity market auction is implemented. Under the net present value framework, the opportunity cost of participating in a capacity market is equal to the expected long-term *Missing Money*, which is equal to the investment fixed cost and the sum of the periodic fixed costs net of the revenue earned on the wholesale and capacity market. This last point is particularly relevant as our paper aims at understanding the link between product design and capacity bids. Therefore, if the costs and profit are held equal, a different product design should bring different long-term *Missing Money*, hence different first bids. We provide a more detailed discussion for a new entrant in the extension section. In the rest of the technical analysis, we focus on bids for existing investments.

4.2. A simple example

We start to illustrate our analysis with an example of bidding behavior with an existing plant. We assume a first product design implementation (case (a)) with a single transaction phase of n_1^t. A second implementation (case (b)) is based on two shorter products of the same length n^t. For simplicity we assume that $n_1^t = 2n^t$. We denote $\bar{T}_1$ the start of the transaction phase for the single product of case (a) and the first product of case (b), and $\bar{T}_2 = \bar{T}_1 + n_1^t$ the start of the second product of case (b). The periodic fixed cost is incurred at $\bar{T}_1$ and $\bar{T}_2$, meaning that we have $n^t = n^{om}$. We denote the total profit collected on the whole period as Π_1, while we denote the profit collected on the first sub-period $\Pi_2^{'}$, and on the second sub-period $\Pi_2^{''}$. Finally, we denote t_0 the date when the auction for the single product of case (a) and the first product of case (b), and $t_0^{'}$ the auction date for the second product of case (b). For any case, the period between an auction and the starting date is equal and noted n^d. We illustrate the two implementations in the following figure.



Following our framework the expected bid at t_0 in the case (a) noted b^1 is equal to:

$$b^1 = e^{-rn^d}\left[\mathbb{E}_0^*\left[c^{om}\int_0^{2n^t} e^{-rt}dt - \int_{n^d}^{2n^t+n^d} \pi_t dt\right]\right]^+$$

Which gives when rearranged:

$$b^1 = e^{-rn^d}\left[c^{om}\int_0^{n^t} e^{-rt}dt + c^{om}\int_0^{n^t} e^{-r(t+n^t)}dt - 2n^t e^{rnd}\pi_0\right]^+$$

While the sum of the two bids in the case(b) noted b^2 is:

$$b^2 = e^{-rn^d}\left[\mathbb{E}_0^*\left[c^{om}\int_0^{n^t} e^{-rt}dt - \int_{n^d}^{n^t+n^d} e^{-rt}\pi_t dt\right]\right]^+ +$$
$$e^{-r(n^t+n^d)}\left[\mathbb{E}_0^*\left[c^{om}\int_0^{n^t} e^{-rt}dt - \int_{i=n^d+n^t}^{2n^t+n^d} e^{-rt}\pi_t dt\right]\right]^+$$

Which gives when rearranged:

$$b^2 = e^{-rn^d}\left(\left[c^{om} - n^t e^{rnd}\pi_0)\right]^+ + \left[c^{om}e^{-rn^t} - n^t e^{rnd}\pi_0)\right]^+\right)$$

The proposition 1 states that the following inequality always holds for any value of the expected inframarginal rent and periodic fixed cost: $b^2 \geq b^1$.

Proposition 1. *Assuming the absence of risk aversion and for an existing investment, a product with a longer transaction phase will always lead to a lower bid than the sum of the bids for products with a shorter transaction phase.*

Proof. The proof is straightforward and is given by the triangle-inequality like of the maximum function: $max(x,0)+max(y,0) \geq max(x+y,0)$. Using the previous example, x takes the value of: $c^{om} - n^t e^{rnd}\pi_0$; and y takes the value of: $c^{om}e^{-rn^t} - n^t e^{rnd}\pi_0$. □

A risk-averse investor could imply a different result. Indeed, a more prolonged transaction phase implies a higher variance than the sum of shorter products' variance. It stems directly from the Geometric Brownian Motion assumption. Therefore, for high-risk-averse investors, having the possibility to bid again could be more profitable if the uncertainty of being retained in future auctions is not significantly high. This proposition holds either from an initial analysis of the bids at date t_0 or reasoning on an expected basis. It is straightforward that the bids and their realized total value can differ from the proposition. For instance, assume that the wholesale price follows a path well above its expected value, then the sum of the bids for lower transaction phases might finally be lower than for a capacity product with a longer transaction phase.

It should be noted that when the closing period differs from the transaction phase, the producers' offer can be significantly affected without impacting the previous proposal. When $n^{om} > n^t$ then the opportunity cost is estimated based on the n^{om} period with the implication that in subsequent auctions, a closing period that overlaps two transaction phases is not taken into account in the offer for the second auction, the opportunity cost being zero because the plant is already open. Similarly, when $n^{om} < n^t$, the opportunity cost is calculated based on a period kn^{om} such that k determines the smallest period greater than the transaction phase ($k \equiv min(kn^{om} - n^t)$ s.t $k \in \mathbb{Z}^+$). When n^t is a multiple of n^{om}, this does not change the offer made by the producer (see, for instance, the previous example). Otherwise it is similar to the case $n^{om} > n^t$. Using these results, we will not study complex cases where the closing periods and transaction phases are not equal in the rest of this paper. While they can explain some actual bidding behavior in volatility and magnitude, they do not change the actual results.

V. THE REAL OPTIONS FRAMEWORK

5.1. A capacity bid as European Put Option

The previous framework provides the marginalist intuition behind the bidding behavior on a capacity market. However, it does not consider all the possible rationales, especially when the transaction phase of a capacity product is associated with irreversible managerial decisions. In this section, we conceptualize the capacity product as a real option that allows the option value to not be available over a closing period to avoid potential losses.

We model the option value using the canonical option pricing theory. Following the most simple case with a transaction over one period t with $n^t = n^{om} = 1$, then the availability decision associated to the capacity product for an existing investment is an European Put Option with a payout profile of $max(c^{om} - \pi_t, 0)$. In this case, the periodic fixed cost can be compared to the strike price of a financial option, and the expected inframarginal rent can be compared to the underlying asset. Following the standard approach and the marginalist assumption, the proposition 2 states the bid on a capacity market is equal to the option value of being available:

Proposition 2. *Given the payout profile associated with the capacity product, a bid noted b^{opt} in an auction set at $t_0 = 0$ for a unique transaction phase at starting n^d periods after the auction and a periodic fixed cost of $c^{om} is$:*

$$b^{opt}(\pi_0, c^{om}) = -\pi_0\phi(z) + e^{-rn^d}(c^{om}\phi(z + \sigma\sqrt{n^d})) \tag{11}$$

$$z := -\frac{\ln[\pi_0] - \ln[c^{om}] + (r + \frac{\sigma^2}{2})n^d}{\sigma\sqrt{n^d}} \tag{12}$$

With ϕ the cumulative distribution function of a standard normal distribution.

Proof. See Appendix □

5.2. A capacity bid as Basket Option

We now expand this approach to the more complex case where a transaction phase covers multiple uncertain inframarginal rents. We start with the first case by hanging up our setup with the financial theory for exotic derivatives. We assume that when the transaction phase is expanded over multiple periods, the European Put Option becomes a modified Basket Option. In finance, a Basket Option is defined by a payoff profile dependent on the value of a portfolio of assets, each following a stochastic process such as a Geometric Brownian Motion which can be correlated or independent. Hence, the availability decision associated with a capacity product is similar to exercising a Basket Option, in the sense that the irrevocable decision to stay open at a date $\bar{T}$

implies collecting each inframarginal rent during the transaction phase, which individually follows a Geometric Brownian Motion. With a Basket Option, its exercise would have meant the collection of the individual stock prices. The primary constraint associated with pricing such option is the absence of closed-form representation of the price since a sum of log-normally distribution random variable is not log-normal. However, we use the well-known approximation stated in Assumption 1 to define an analytic approximation of the option price. It allows deriving the following proposition regarding the bid on a capacity market when the transaction phase covers multiple inframarginal rent periods.

Proposition 3. *Given the payout profile associated with the capacity product, a bid noted b^{opt} in an auction set at $t_0 = 0$ for a transaction phase starting of length n^t, starting n^d periods after the auction is set and with a equal closing period of n^t is:*

$$b^{opt}(\pi_0, c^{om}) = -\pi_0 n^t \phi(z) + C^{om}\phi(z+v) \tag{13}$$

$$z = -\frac{m - \ln\left[c^{om}\int_0^{n^t} e^{-rt}dt\right] + v^2}{v}$$

$$C^{om} = e^{-rn^d}c^{om}\int_0^{n^t} e^{-rt}dt \tag{14}$$

With ϕ the cumulative distribution function of a standard normal distribution , m and v^2 defined as follow:

$$m = 2\ln\left[\pi_0\int_0^{n^t} e^{r(i+n^d)}dt\right] - 0.5\ln\left[\pi_0^2\int_0^{n^t}\int_0^{n^t} e^{r(t+s+n^d)+(t+n^d)\sigma^2}dtds\right]$$

$$v^2 = \ln\left[\pi_0^2\int_0^{n^t}\int_0^{n^t} e^{r(t+s+n^d)+(t+n^d)\sigma^2}dtds\right] - 2\ln\left[\pi_0\int_0^{n^t} e^{r(i+n^d)}dt\right]$$

Proof. See Appendix □

This definition of a bid on a capacity market can be understood as follows. First, note that the inframarginal rent term $\pi_0 n^t$ is linked to the Geometric Brownian Motion and the risk-free version of the inframarginal rent process[7]. The fixed cost term is the sum of the actualized periodic fixed costs associated with the decision to stay open. Finally, the value z and $z+v$ are linked with Assumption 1, with the first term in the logarithm of m (second term in v^2) represents the mean of the sum of the inframarginal rent, and

[7]Under the equivalent martingale measure $\mathbb{Q}$, the drift of the inframarginal rent is equal to the risk-free rate, meaning that any actualized expected value of the rent is equal to its initial value π_0.

the second term in the logarithm of m (first term in v^2) is the second moment of the sum of the inframarginal rent.

The extension to the case of a transaction phase covering different closing periods is straightforward. When $n^{om} > n^t$, then the implicit phase during which the option value is estimated is n^{om}. Similar to the net present value case, having a longer duration for the closing period than the implemented transaction is similar to having an implicit transaction phase of the same duration as the closing period. In the case of $n^{om} < n^t$, then we can define the new strike price as the sum of the expected periodic fixed cost incurred as soon as the transaction phase has begun and continued until its end.

5.3. The sum of expected capacity bids

Once we derive the bid for a single product, we can now analyze the sum of the bids for multiple capacity products. Assumption 2 states that even though some correlation exists between inframarginal rents over the investment lifetime, the decision to close during one period does not modify the rent value in a subsequent period. It allows defining the sum of the bids for multiple capacity products as the sum of the value of their options estimated at a single date; for simplicity, here, the auction at which the longer product is sold or the first shorter product is sold.

Proposition 4. *Given the payout profile associated with a capacity product of length $\frac{n^t}{k}$ with the same closing period of $\frac{n^t}{k}$, the sum of expected bids noted b^{opt} made during k successive auctions is:*

$$b^{opt}(\pi_0, c^{om}) = -\pi_0 \frac{n^t}{k} \sum_{j=1}^{k} \phi(z_j)dt + c^{om} e^{-rn^d} \int_0^{\frac{n^t}{k}} e^{-rt}dt \sum_{j=1}^{k} e^{-r(j-1)\frac{n^t}{k}} \phi(z_j + v_j)$$

With:

$$z_j = -\frac{m_j - \ln\left[c^{om} \int_0^{\frac{n^t}{k}} e^{-rt}\right] + v_j^2}{v_j}$$

With m_j and v_j^2 defined as follow:

$$m_j = 2\ln\left[\pi_0 \int_{n^t(j-1)}^{n^t j} e^{r(t+n^d)}dt\right] - 0.5\ln\left[\pi_0^2 \int_{n^t(j-1)}^{n^t j} \int_{n^t(j-1)}^{n^t j} e^{r(s+t+n^d)+(t+n^d)\sigma^2} dtds\right] \quad (15)$$

$$v_j^2 = \ln\left[\pi_0^2 \int_{n^t(j-1)}^{n^t j} \int_{n^t(j-1)}^{n^t j} e^{r(s+t+n^d)+(t+n^d)\sigma^2} dtds\right] - 2\ln\left[\pi_0 \int_{n^t(j-1)}^{n^t j} e^{r(t+n^d)}dt\right] \quad (16)$$

Proof. With no correlation between the decision to close and the profits made during other periods, the different capacity products could be conceptualized as options on different assets (See Trigeorgis 1993). □

VI. COMPARATIVE STATISTICS ON THE CAPACITY BIDS

Using the results of the previous section, we compare the bidding behavior in a capacity market depending on: (i) the drivers behind the opportunity cost, (ii) if producers follow the net present value framework or the real options framework, (iii) the design of the transaction phase for a given period.

We start by studying the evolution of the bids under the real options framework, especially concerning the length of the product. As our framework is new to this topic, we also describe the effect of the main variables on the bid. This step is particularly relevant due to the length of the transaction phase, which significantly complexifies the analysis. Then, we show how the real option bidders value the possibility of closing to avoid the fixed costs. Namely, we describe the flexibility associated with the option to close and how its value is impacted by the length of the transaction phase and other variables. Finally, we discuss the impact of segmenting a capacity product in multiple shorter transaction phases. In a similar fashion as the two previous analyses, we also provide the impact of the drivers on the delta for the bids between different capacity product designs.

6.1. The bids value

This analysis sheds light on how a capacity bid varies with its fundamentals. One of the critical variables in this research is the length of the product n^t. We also look at the two main variables of the opportunity cost as described in the net present value section, namely the initial value of the inframarginal rent π_0 and the fixed cost value c^{om}. Next, we study the regulatory parameters chosen when the capacity market is implemented n^d, representing the waiting time before the transaction phase. Although, we left for future work a deeper analysis of this parameter on market efficiency [8], our analysis still provides some insight on its role in the bidding behavior on the capacity market. Finally, we also study the impact of different volatility levels of the inframarginal rent σ on the capacity bid. Indeed, one of the current policies in the power sector relies on significantly increasing the share of renewables in the production mix. A key consequence would be an immediate increase in the volatility of the wholesale price [16]. In turn, it also has an indirect effect on the bidding behavior in the capacity market[9].

For relevancy and simplicity, we assume that the periodic fixed cost occurs simultaneously as the inframarginal rent in this section, namely every t period. Therefore, when the transaction phase increases by one period, the producer gains an uncertain infra-

[8] The waiting time has been numerously cited as a critical regulatory parameter when designing capacity markets. However, to our knowledge, very few papers have looked into this issue from a modeling perspective.

[9] A second indirect effect is the decrease of the average wholesale price due to a merit order effect, which translates into a lower initial value π_0.

marginal rent and sustains an additional unitary periodic fixed cost. We provide in proposition 5 an overview of the effect of increasing the transaction phase on the bids for the net present value case and the real option case. Note that for the real options framework, we could not find a closed-form solution in terms of the value of the variables for which a clear-cut answer on the sign of the derivatives exists. However, we can provide sufficient general conditions so that such a solution exists. We also show that for extreme values of n^t, the derivative of the bid with respect to n^t is always positive. This result tends to confirm that an increase of n^t increases the capacity bid on the capacity market under a real options framework, which is not necessarily the case under the net present value framework. The conditions presented in this section are usually given on the derivatives of the variable z, which represents the threshold on the probability that the sum of the actualized fixed cost (the strike price) is above the sum of uncertain inframarginal rent (the underlying asset), and on specific ratios between the density functions $\phi(z)$ and $\phi(z+v)$, which express the probability of a standard normal distribution at their respective value[10].

Proposition 5. ***NPV**: The bid is a concave function with respect to the length of the transaction phase n^t. The threshold in term of fixed cost between an increasing bid and a decreasing bid is given by: $c^{om} = \pi_0 e^{r(nd+nt)}$.*

***RO**: An increase in the transaction phase increases the capacity bid if the following conditions holds:*

- *Fixed costs: $c^{com} \int_0^{n^t} e^{-rt}dt \geq \sqrt{\pi_0} V e^{v^2(2r\frac{\partial v^2}{\partial n^t}-1)}$*
- *Cdf ratio: $\frac{S_1 C^{om}}{\pi_0} \geq R_0 = \frac{\phi(z)}{\phi(z+v)}$*
- *Df ratio: $\frac{C^{om}}{n^t \pi_0} \geq R_1 = \frac{\varphi(z)}{\varphi(z+v)}$*

with C^{om} the actualised sum of the fixed cost and $S_1 = e^{-rn^t} / \int_0^{n^t} e^{-rt}dt$.

Moreover, when $n^t \to +\infty$ then the derivative of the capacity bid with respect to n^t converges toward to the marginal change of the expected fixed cost equal to $c^{om}e^{-r(n^t+n^d)}$ which is always positive. When $n^t \to 0$, then the derivative of the capacity bid with respect to n^t converges either toward 0 or toward a positive value equal to $c^{om}(e^{-r(n^t+n^d)} - \pi_0)\phi(z)$

Proof. See Appendix □

The intuition behind this result is as follows. For the net present value bid, it is straightforward because the bids are equal to $\pi_0 n^t - C^{om}$. Therefore the bid starts decreasing as soon as the marginal value of the expected profit at the end of the transaction phase exceeds the additional fixed marginal cost.

For the option value, recall that it is composed of two distinct parts[11]: (i) a negative part which stands for the expected sum of inframarginal rent adjusted by the cumu-

[10] Recall that $\phi'(z) = \varphi(z)$.

[11] The derivative of the bid value with respect to n^t has the following form: $\frac{\partial b^{opt}}{\partial n^t} = -\pi_0(\phi(z) + n^t \frac{\partial z}{\partial n^t}\varphi(z)) + C^{om}(S_1\phi(z+v) + \frac{\partial z+v}{\partial n^t})\varphi(z+v))$.

lative distribution function of the standardised normal distribution: $-\pi_0 n^t \phi(z)$; (ii) a positive part which stands for the strike price represented through the cost of staying available, also adjusted by the cumulative distribution function: $C^{om}\phi(z+v)$; with C^{om} the actualised sum of the fixed cost. For both parts, an increase of n^t has two effects: a direct effect, linked to the marginal increase in the profit and the fixed cost; and an indirect effect via a change in cumulative distribution function value. The sign of this indirect effect depends on the sign of the derivatives of z and $z+v$ with respect to n^t. We analyze now how the conditions can be widened or tightened to have a clear-cut effect on the sign of n^t on the bid.

The first effect is straightforward: an increase in n^t also increases the profit and fixed costs. For the indirect effect, we start with the fixed cost part. Recall that the cumulative distribution function represents the the probability that the periodic fixed cost is above the inframarginal rent (ie. $F_{nt\pi_0}(C^{om}) = \phi(z+v)$). The sign of the corresponding derivative $\frac{\partial z+v}{\partial n^t}$ is positive only if the following condition on the fixed cost holds: $c^{com}\int_0^{n^t} e^{-rt}dt = \sqrt{\pi_0}Ve^{-2rv^2\frac{\partial v^2}{\partial n^t}}$, with V^2 second moment of the distribution of the inframarginal rent sum during the transaction phase. [12] In this case, an increase of n^t always implies an increase of the probability that the sum of the actualized fixed cost is above the sum of uncertain inframarginal rent. Hence, both effects of the part (ii) are always positive, implying that an increase of n^t increases the option value.

However, the effect can be ambiguous for some value of n^t due to part(i). The sign of the corresponding derivative $\frac{\partial z}{\partial n^t}$ is negative only if the following condition on the fixed cost holds: $c^{com}\int_0^{n^t} e^{-rt}dt = \sqrt{\pi_0}Ve^{v^2(2r\frac{\partial v^2}{\partial n^t}-1)}$. When the derivative is negative it decreases the probability that the inframarginal rent is above the fixed cost given by $\phi(z)$, (recall that $\mathbb{E}\left[n^t\pi_0 | n^t\pi_0 < C^{om}\right] = n^t\pi_0\phi(z)$), hence it lowers the bid value. This value is decreasing with n^t. Therefore, the possibility that the bid is positively impacted by n^t increases with n^t.[13]

The last part of the proposition supports the results that the effect of n^t is almost surely positive on the bid value. This result relies on the evolution of the distribution function and the cumulative distribution function φ and ϕ with respect to n^t. When n^t converges toward 0 or $+\infty$, the evolution of threshold values z and $z+v$ implies that the function φ converges towards 0 is consistent with a normal distribution. For the cumulative distribution function, we use first the two conditions on the fixed costs which determines the sign of $\frac{\partial z}{\partial n^t}$ and of $\frac{\partial z+v}{\partial n^t}$ [14]. the first condition converges toward 0 as n^t, implying that n^t always decreases z. Therefore, $\phi(z)$ converges toward 0 as n^t increases, which means that an increase in the length of the transaction phase always decreases the probabil-

[12]This is a weaker condition so that $\frac{C^{om}}{n^t\pi_0} \geq R_0$ which implies a positive effect of n^t on the bid. Note that this threshold is always increasing with n^t.

[13]Note that the ratios R_0 and R_1 are similar to other ratio that can be found in the model. For instance, the constrain that the option value is always positive implies that we have the following condition:

$$\frac{C^{om}}{n^t\pi_0} \geq \frac{\phi(z)}{\phi(z+v)}$$

[14]Recall their respective values: $c^{com}\int_0^{n^t} e^{-rt}dt = \sqrt{\pi_0}Ve^{v^2(2r\frac{\partial v^2}{\partial n^t}-1)}$ and $c^{com}\int_0^{n^t} e^{-rt}dt = \sqrt{\pi_0}Ve^{-2rv^2\frac{\partial v^2}{\partial n^t}}$.

ity that the inframarginal rent is above the period fixed cost. On the other hand, the second condition converges toward $+\infty$ when n^t increases[15] This implies that $\phi(z+v)$ converges towards 1, as a reverse effect of $\phi(z)$. Finally, when n^t converges towards 0, the volatility of the sum of inframarginal rent converges towards 0, which implies that $\phi(z+v)$ converges toward $\phi(z)$. When the sign $\frac{\partial z}{\partial n^t}$ is negative, then $\phi(z)$ converges towards 0 as n^t decreases, which proves the first convergence. When the sign $\frac{\partial z}{\partial n^t}$ is positive, then the relation between the value of π_0 and the moments of the sum of the inframarginal rent gives the second convergence.

We turn now to the other drivers for the capacity bid, and we provide the result in lemma 1. Again, as there are no closed-formed solutions, we provide conditions for which the drivers have clear-cut signs on the bid value.

Lemma 1. *The value of the bid under real option:*

1. *decreases with the inframarginal rent* π_0
2. *increases with the periodic fixed cost* c^{om}
3. *is ambiguous with the waiting time* n^d
4. *is ambiguous with the inframarginal rent volatility* σ

Result (1) and (2) holds if the condition $\frac{C^{om}}{n^t\pi_0} \geq R_1$ *is satisfied.*

For Result (3), n^d *always decreases the bid when* $c^{om}\int_0^{n^t} e^{-rt}dt \geq Ve^{-M\frac{r+\sigma}{r-\sigma}}$, $\frac{C^{om}}{n^t\pi_0} \geq R_1$ *and* $r > \sigma$ *are satisfied. If only the first condition holds and that* $-r\phi(z+v) \geq \frac{\partial v}{\partial n^d}\varphi(z+v)$*, then* n^d *always decreases the bid.*

For Result (3), n^d *always increases the bid when* $c^{om}\int_0^{n^t} e^{-rt}dt \geq Ve^{-M\frac{r+\sigma}{r-\sigma}}$ *and* $\frac{\partial v}{\partial n^d}\varphi(z+v) \geq -r\phi(z+v)$*. Otherwise* n^d *has an undetermined effect on the bid price and depend on the relation between* $\frac{C^{om}}{n^t\pi_0}$ *and the ratio* $R_3 = \frac{\frac{\partial z}{\partial n^d}\phi(z)}{-r\phi(z+v)+\frac{\partial v}{\partial n^d}\varphi(z+v)}$

For Results (4), σ *always increases the bid if* $c^{om}\int_0^{n^t} e^{-rt}ft \leq \frac{M^2}{V}$ *and* $\frac{C^{om}}{n^t\pi_0} \geq R_1$ *are satisfied. Otherwise if* $c^{om}\int_0^{n^t} \leq V$*, then* σ *always increases the bid if* $\frac{C^{om}}{n^t\pi_0} \geq \frac{\varphi(z)\frac{\partial z}{\partial \sigma}}{\varphi(z+v)\frac{\partial z+v}{\partial \sigma}}$*. If the conditions are not respected then* σ *is always decreasing the bid.*

Proof. See Appendix □

The first and second results are standard regarding real options theory (see, for instance, [27]). They have an opposite interpretation: an increase of the initial inframarginal rent signal that future revenues will also increase. Consequently, it decreases the value to close to avoid the fixed cost, as those costs are more likely to be covered by the inframarginal rent. On the other hand, as the fixed cost increase, then the value increases.

[15]Which is consistent with the fact that the volatility of the sum of inframarginal rent v increases with n^t.

The analysis of n^d is less intuitive and relies on stronger conditions on the fixed cost.[16] The effect of n^d is unique on the inframarginal rent part of the bid and depends on the sign of the corresponding derivative. If the derivative of this part $\frac{\partial z}{\partial n^d}$ is negative, it implies that if n^d increases, then the probability that the rent is above the fixed cost decreases. Otherwise, the reverse effect happens. It is given by the first condition on the fixed costs for n^d to have a negative effect on the bid. To highlight the effect of n^d on the fixed cost part of the option value we rearrange the conditions of result (3) which gives: $C^{om}(\varphi(v+v)\frac{\partial v}{\partial n^d} - r\phi(z+v)) + \frac{\partial z}{\partial n^d}\Delta\varphi$ with $\Delta\varphi = (C^{om}\varphi(z+v) - n^t\pi_0\varphi(z))$. This value is central in our analysis, and it is always positive as long as $\frac{C^{om}}{n^t\pi_0} \geq R_1$ holds and can be found in the four derivatives of the bid value. It gives the second condition for n^d to have a negative effect on the bid. The derivative represents the net marginal change in terms of the bid value when the variables marginally impact z.

We turn now to the first part of the rearranges derivative.[17] When n^d increases, it always decreases the value of the fixed cost because of a discounting effect (second negative term), while also changing the volatility of the revenue (first term). Following the analysis of $\frac{\partial z}{\partial n^d}$, it is sufficient for its sign to be negative and also to have a negative sign $\frac{\partial \sigma}{\partial n^d}$ to have the decreasing effect of n^d on the bid. It has straightforward intuitions: if an increase of n^d decreases the value of the fixed costs, the volatility, and the probability that the inframarginal rent is below the fixed costs, then the option value also decreases, hence the bid. The derivative in the first term is negative if and only if $\frac{\partial v}{\partial n^d}$ is also negative. That is, an increase of n^d decreases the volatility of the total revenue. It is the case only when the risk-free rate (r) is above the volatility of the inframarginal rent (σ). It gives the third condition for n^d to have a negative effect on the bid. Therefore the first group of conditions for n^d simply states the conditions under which every part of the derivative is negative.

Finally, when at least one derivative is not negative, then the sign is ambiguous and depends on the magnitude of each part of the derivative. For instance, when $\frac{\partial z}{\partial n^d}$ is still negative, but an increase of n^d decreases the volatility of the sum of inframarginal rent (i.e., $\frac{\partial v}{\partial n^d}$ is positive), it is sufficient that the negative effect of the inframarginal rent is higher than the gains in terms of volatility to ensure that the sign of n^d is negative. Therefore it gives the second set of conditions. The opposite case when $\frac{\partial z}{\partial n^d}$ is positive gives the third set of conditions.

To conclude, we state that the effect of σ is also counter-intuitive when the conditions do not hold. Again, we rearrange the derivative which gives[18]: $C^{om}\varphi(v+v)\frac{\partial v}{\partial \sigma} + \frac{\partial z}{\partial \sigma}\Delta\varphi$. First, note that the first derivative is positive as an increase of the volatility of the periodic revenue always increases the volatility of the total revenue. Therefore the ambiguity of σ on the bid value only depends on the sign of the second derivative $\frac{\partial z}{\partial \sigma}$, and on the magnitude of the positive parts. When analyzing the sign of the derivative ($\frac{\partial z}{\partial \sigma}$), we find that it requires one condition to be negative: $c^{om}\int_0^{n^t} e^{-rt}ft \geq \frac{M^2}{V}$. Recall that M^2 and V^2 are the first and second moments of the distribution of the inframarginal

[16]The derivative of the bid value with respect to n^d is: $-\pi_0 n^t \frac{\partial z}{\partial n^d}\varphi(z) + C^{om}(-r\phi(z+v) + \frac{\partial z+v}{\partial n^d}\varphi(z+v)$.

[17]Note the absence of any inframarginal rent in this part due to the Geometric Brownian Motion assumption where the rate of increase of the inframarginal rent is equal to the risk-free rate.

[18]The derivative of the bid value with respect to σ is: $-\pi_0 n^t \frac{z}{\sigma}\phi(z) + C^{om}\frac{\partial z+v}{\partial \sigma}$.

rent sum during the transaction phase.[19] Such a condition highlights the key role of the difference between the fixed costs and the expected value and distribution of the inframarginal rent. When the fixed costs are relatively high compared to the mean value adjusted by the risk of the total revenue (i.e., the conditions is satisfied), then a marginal increase of the volatility always implies a loss for the option value: it increases the occurrence of having the sum of inframarginal being above.

6.2. The value of flexibility

We turn to the analysis of the difference between a bid under a net present value framework and a bid under a real options framework. As in the canonical real options theory, the possibility of the managerial option always creates additional value for the producers. Consequently, when comparing the difference between bidding the missing money and bidding the option value associated with the possibility to close, we have the following proposition.

Proposition 6. *Under the same market design, the bid in a capacity market when producers consider the option value is always higher or equal to the bid using only a net present value approach.*

$$b^{opt} \geq b^{npv} \tag{17}$$

The proof for the proposition is straightforward and comes from the definition of an net present value and a real option bid. Under the first framework, producers bid the maximum of their expected Missing Money: $b^{npv} = e^{-rn^d}\left[\mathbb{E}_0^*\left[C^{om} - \int_0^{n^t} e^{-rt}(\pi_t)dt\right]\right]^+$. On the other hand, under the second framework, producers bid their option value, namely the expected maximum of their missing money: $b^{opt} = e^{-rn^d}\mathbb{E}_0^*\left[\left[C^{om} - \int_0^{n^t} e^{-rt}(\pi_t)dt\right]^+\right]$. Therefore we always have $b^{opt} \geq b^{npv}$. Using this result we can now analyse the value of the flexibility which is the difference between the two bids. We define Γ as the value of the flexibility such as: $\Gamma = b^{opt} - b^{npv}$. It is defined in the following equation:

$$\Gamma = -\pi_0 n^t(\phi(z) - 1) + C^{om}(\phi(z+v) - 1) \tag{18}$$

The analysis of the evolution of Γ with respect to the main variables is similar to the comparative statistics made in the previous section. Indeed, for any variable x note that $\frac{\partial \Gamma}{\partial x} = \frac{\partial b^{opt}}{\partial x} - \frac{\partial b^{npv}}{\partial x}$. Therefore, the net effect of the previous results can either be increased or decreased depending on how the bid under the net present value framework behaves. We summarise the main results in proposition 7.

Proposition 7. *The length of the transaction phase has an ambiguous effect on the value of the flexibility:*

- Γ *is increasing in* n^t *when* $c^{om} \leq \pi_0 e^{r(nd+nt)}$

[19] Note the resemblance with the Sharp Ratio used in finance which defines the performance of an investment compared to a risk-free asset, after adjusting for its risk.

When the condition does not hold, given the following ratio:

$$R_4 = \frac{\frac{1}{n^t}\phi(z) + \frac{\partial z}{\partial n^t}\varphi(z) - \frac{1}{n^t}}{S_1\phi(z+v) + \frac{\partial z+v}{\partial nt}\varphi(z+v) - S_1} \tag{19}$$

- Γ *is increasing in* n^t *if* $S_1(\phi(z+v)-1) + \frac{\partial z+v}{\partial nt}\varphi(z+v) > 0$ *and the following condition holds:* $\frac{C^{om}}{n^t\pi_0} \leq R_4$
- *Otherwise,* Γ *is increasing in* n^t *if* $S_1(\phi(z+v)-1) + \frac{\partial z+v}{\partial nt}\varphi(z+v) < 0$ *and the following condition holds:* $\frac{C^{om}}{n^t\pi_0} \geq R_4$.
- *When the conditions do not hold,* Γ *is decreasing in* n^t

Proof. The proof is similar to the Proposition 5, the results follow directly from the derivative of the bid with respect to the variable. □

The ambiguity of n^t on the value of flexibility comes from the concavity of the net present value bid as described by the proposition 5 and monotonicity of the real option bid. When the former decreases with respect to n^t, the flexibility is always increasing as the latter increases. However, when the net present value bid increases, the sign of the flexibility derivative depends on the magnitude between a marginal increase of the net present value bids and the marginal increase of the real option bids. Those opposing effects are materialized in the ratio R_4 with the negative terms $-\frac{1}{n^t}$ and $-S_1$, which represent the marginal increase of respectively the profit and the fixed cost part. Given the results expressed in proposition 5 and proposition 7, we find that the condition under which the flexibility increases with respect to n^t is weaker than the condition under which the real option bid increases with respect to n^t.

This ambiguity renders significantly complex the analysis of the value of flexibility with respect to other drivers. While, we had intuitive results and mild conditions for the two fundamental drivers of the bids in proposition 5, namely π_0 and c^{om}. The marginal increase of the net present value bids is always superior to the direct effect observed in the real option bids when the two drivers increase. For instance, recall that an increase of π_0 both directly lower the value of the option due to a marginal decrease of $-n^t\phi(v)$, while indirectly modifying the option value with a change in the probability of the maximum function: $-\pi_0 n^t \frac{\partial z}{\partial \pi_0}\varphi(v)$. When we introduce the change in the net present value bids, the marginal decrease when π_0 is equal to $-n^t$. It implies that we always have: $n^t \geq n^t\phi(v)$, as $\phi(v)$ is a cumulative density function. Therefore, the net effect on the value of the flexibility is always going to be dependant on the magnitude between the net direct effect $n^t(1-\phi(v))$ and $-\pi_0 n^t \frac{\partial z}{\partial \pi_0}\varphi(v)$, and not anymore on the sign of the derivative.

However, for the volatility of the inframarginal rent, we find a strict identical effect between the real option bid and the flexibility value. Indeed, the net present value bid is independent of the volatility , hence $\frac{\partial \Gamma}{\partial \sigma} = \frac{\partial \pi_0}{\partial \sigma}$. Therefore, the sign and the conditions discussed in lemma 1 can be applied to the analysis of the value of the flexibility with respect to σ.

6.3. Bids and product design

In this section, we analyze the effect of segmenting the capacity product into multiple shorter products. More precisely, we assess the difference between the bid for a single capacity auction considered as a "long" product covering a transaction phase of n^t periods with the sum of expected bids for k successive capacity auctions where producers can sell "short" product of length $\frac{n^t}{k}$. The difference in the cost of a capacity market, noted Δb^{opt}, given this configuration is defined in the following equation and uses the proposition 3 and proposition 4:

$$\Delta b^{opt} = -\pi_{t_0} n^t \left(\phi(z) - \frac{1}{k} \sum_{i=1}^{k} \phi(z_i) \right) + C^{om} \left(\phi(z+v) - S_2 \sum_{i=1}^{k} e^{-r(i-1)\frac{n^t}{k}} \phi(z_i + v_i) \right) \tag{20}$$

With $S_2 = \frac{\int_0^{\frac{n^t}{k}} e^{-rt} ft}{\int_0^{n^t} e^{-rt} ft}$.

This equation is key to understanding different product designs' impact on bidding behavior under the real options framework. Indeed note that both for the profit and the fixed cost part of the difference, the equation shows that the sign of Δb^{opt} depends on the relation between the cumulative distribution function of the longest product ($\phi(z)$ and $\phi(z+v)$) with an average value of the cumulative distribution function of the shortest product. This average is shown directly with $\frac{1}{k}$ or indirectly with the value of S_2, which takes into account the discounting effect of the periodic fixed cost. Therefore, it is sufficient for the average effect for the profit part to dominate (resp. to be dominated) $\phi(z)$ and the second average effect to be dominated (res. dominate) $\phi(z+v)$ to have an increase (resp. decrease) of the bid when segmenting the capacity product into a shorter product. Again we do not have a closed-form solution that allows guaranteeing a value for the variables to give a clear-cut answer on the sign of this difference. However, we provide in lemma 2 sufficient conditions that allow such a clear-cut answer to exist.

Lemma 2. *The sum of expected bids of shorter products is always lower or equal to the individual bid for the longer product when each threshold z_i is lower or equal to the unique threshold z, and that each threshold $z_i + v_i$ is above or equal to the unique threshold $z + v$.*

Proof. The proof is straightforward and stems from the definition of ϕ as the cumulative density function of a standard normal distribution □

Those conditions imply that the probabilities (and the average probability) that the sum of inframarginal rent is below the fixed cost (i.e., $\phi(z+v)$ and $\phi(z_i+v_i)$) are consistently higher under the product design with short term products. Naturally, this condition states that it should also decrease the curtailed expected value given in part by the expressions $\phi(z)$ and $\phi(z_i)$ for the shorter period product design.

From the definition of Δb^{opt}, it is easy to deduce its marginal change with respect to a change of the main drivers. The comparative statistics on the difference between two market designs encompass both the analysis provided in the first section on the value of the bid and the average component present in equation 20. When we derive this value with respect to the set of variables (namely, π_0 c^{om} n^d and σ), we find that it depends again on some conditions on the fixed costs and the sign of the derivatives. More precisely, it relies on the difference between the marginal effect of the first section on the average marginal effect of the sum of expected bids. Both elements can be analyzed separately using the results in lemma 1.[20] To see this, we provide in the following equation the marginal change of Δb^{opt} with respect to the volatility of the inframarginal rent σ:

$$\begin{aligned}\frac{\partial \Delta b^{opt}}{\partial \sigma} = -\pi_0 n^t \left(\frac{\partial z}{\partial \sigma}\varphi(z) - \frac{1}{k}\sum_{1}^{k} \frac{\partial z_i}{\partial \sigma}\varphi(z_i) \right) + \\ C^{om} \left(\frac{\partial z + v}{\partial \sigma}\varphi(z+v) - S\sum_{i=1}^{k} \frac{\partial z_i + v_i}{\partial \sigma}\varphi(z_i) \right)\end{aligned} \tag{21}$$

Therefore, an increase in the volatility of the inframarginal rent positively increases the difference between the bid for the longer product and the sum of the expected bid for shorter products if and only if the average effect of the latter is above (resp. below) the marginal change of the former for the profit part (resp. periodic fixed cost) of the option value.

VII. CASE STUDY

We illustrate our model by simulating a power plant participating in the French capacity market. This mechanism has been recently implemented, with a first auction held in 2016 for a transaction phase starting in 2017. The main characteristics regarding the supply side rely on a 4-year quasi-continuous forward market. Each capacity product covers a year, with an obligation of being available concentrated between January and March. They can be traded up to four years before the delivery year, either through multiple auctions or bilateral tradings. Figure 1 shows the clearing price in the French capacity market for the corresponding yearly period of the transaction phase and each auction before the starting date. Excluding the specific 2017 and 2018 period, the capacity price is on average equal to 23 191 € /MW, with a maximum value of 47 400 € /MW and a minimum value of 13 000 € /MW. We use the average price for the transaction phase in 2022 as a comparative basis when simulating the output using the previous results. The average price is equal to 25 314 € /MW.

We consider an investment in a CCGT gas power plant for which the lifetime is 30 years, and we normalize the capacity to 1MW. The production costs have been taken from the consultation report made by RTE, the transport and system operator, which had to build the rules for the capacity market. For an existing power plant, the fixed

[20] Each element of the sum for shorter products are independent, which allows adding the marginal change of each element.

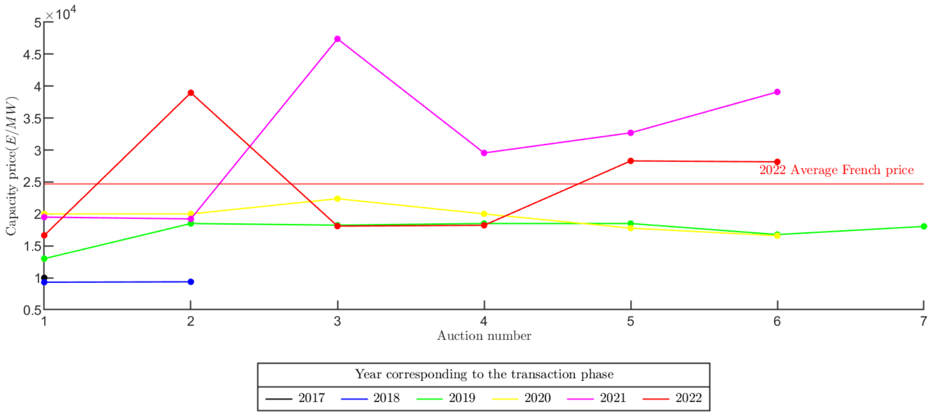


Figure 1: Auction results for the French capacity market

operating cost is equal to 32.5€ /kMW.yr,[21] which translates into a periodic fixed cost value of 98.63 € /MW.day. The variable production cost includes the fuel cost and the carbon costs and is equal to 25 € /MWh. We consider them fixed during the lifetime of the investment.

For the French system, we assume the risk-free yearly rate is 2.32 %. It is the average interest rate of the 30-year government bonds for France between the years 2009 - 2021. It implies a daily value of 0.64%. Then, we estimate the stochastic process. First, we analyze the forward Y1 traded on the French power exchange between the years 2010 - 2015. We find that the average daily electricity price over the period is equal to 47.15€ /MWh, with a maximum value of 61.65 € /MWh and a minimum value of 33.50 € /MWh. A gas power plant can be considered peak technology or semi-peak technology in the French system, and it does not receive an inframarginal rent every hour during its lifetime. Therefore, we first compute an average price duration curve which gives the proportion of time for which the price exceeded a specific value. Then, we use the data on the marginality duration of a gas power plant given by the yearly report *Functioning of the wholesale electricity* from the CRE, the French electricity regulator. We find that on average such investment is either the marginal or an inframarginal bidder for 57% of the time in a year, with a high deviation between years ranging from 5% to 85% over the years 2010 - 2019. Given this significant range, and using the marginal production cost and the price duration curve, we find an interval of daily inframarginal rents from 59 € /day and 949 € /day with a mean value of 641 € /day for an investment selling all the time.[22] We acknowledge this value is highly uncertain and dependent on the actual plant. Therefore, we use those values as a comparative order of magnitude rather than real input [23]. For the volatility of the inframarginal rent, we use the for-

[21] It is the mean value for a range between 30 and 35 € /kMW.yr.

[22] Therefore, we also consider the variable π_0 as the average inframarginal rent being available all the time on the wholesale market.

[23] A study made by RTE has found that between 2010 - 2018, the annual inframarginal rent for a CCGT ranges daily from 28 € /MW to 188 € /MW. The numerical simulation gives the same order of magnitude with respect to the initial value π_0.

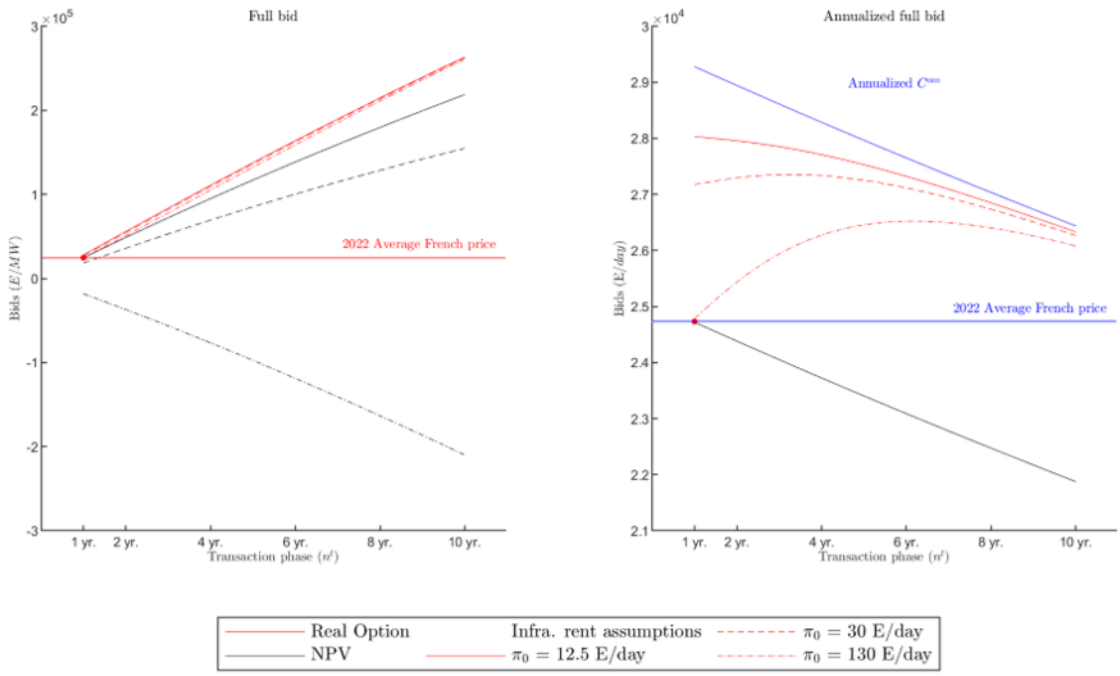


Figure 2: Single capacity bid for different transaction phases value

ward Y1 traded on the French power exchange between the years 2010 - 2015. We find a volatility value of 0.00578, close to the range used in [16] for the Italian market.[24]

7.1. Bids in a capacity market under the net present value and the real options framework

We first analyze how the bid in a capacity market can vary with respect to the length of the transaction phase. We provide the results both under the net present value framework and the real options framework. Using the French capacity market as our reference design, we use an initial value for the transaction phase (n^t) of one year with a waiting phase (n^d) of four years. The results of the simulation are presented in figure 2.

As shown in proposition 5, a capacity bid always increases with the length of the transaction phase under the real options framework. On the other hand, the net present value bid can decrease for relatively high values of the inframarginal rent. For the initial values of n^d and n^t, beyond an initial value of 87.83 € /MWh, the bid is constantly decreasing with respect to n^t. The figure also shows that for a given initial value of the inframarginal rent π_0, the bid under the real options framework is always above the bid under the net present value framework, as shown in proposition 6. We also provide the annualized value of the capacity bid for more clarity. We show in the second figure the corresponding hypothesis for the two frameworks that leads to the same bid as the average one observed in the French Capacity market for the transaction phase of the year 2022. Under the net present value framework, the initial value for the inframarginal rent is equal to 12.5 € /MWh, which is significantly below the lower range

[24]Note that the value of the volatility is consistent with the threshold found in [25] regarding the approximation condition of Assumption A .

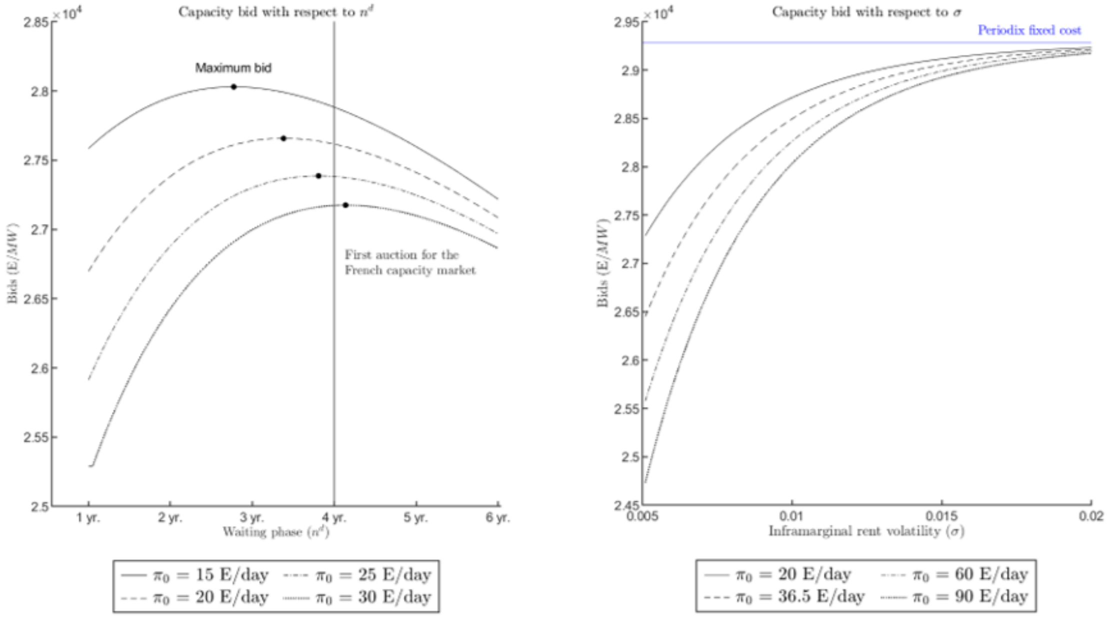


Figure 3: Comparative statistics on the capacity bid

of the value found using the forward data. It corresponds to being marginal only 1% during a year on the electricity market. Under the real options framework, the initial value is 130 € /MW, almost ten times higher than the previous value. This high value is within the range of the marginality and corresponds to be marginal at least 11% of the time. Note that the capacity price under the real options framework converges towards the periodic fixed cost as expected when analyzing the derivative of the capacity bid. Therefore, the real options framework always converges toward the canonical behavior of a capacity market. That is, the bids should be equal to the fixed costs. Finally, the data used in the numerical illustration shows that the real option bids tend to increase more rapidly than the net present value bids, which implies that the flexibility increases concerning the length of the transaction phase.

7.2. The effect of the waiting time and the volatility on the capacity bid

We analyze in this section how the bid on a capacity market can be modified by choosing a policy instrument, namely the waiting time between the capacity auction and the start of the transaction phase, and by the volatility of the inframarginal rent, a key variable to understand industrial decisions in the power sector. Figure 3 provides the result of our numerical simulation

Regarding the waiting time, we underline the ambiguous effect of this variable on the capacity bid. Beyond a specific value of n^d represented by the black dots on the first figure, an increase of the value of n^d continuously decreases the capacity bids. Below this value, the waiting time always increases the capacity bid. This threshold depends on the assumption concerning the initial value of the inframarginal rent. A higher initial value implies a higher threshold. To say it differently, when the producer forecasts a more profitable investment, it decreases the capacity bid and reduces the potential negative effect of n^d on the bid. The reason for such results is as follows. First, given the initial data, the bid part relative to the inframarginal rent is always positively impacted by n^t. It means that the sign of the derivative of z with respect to n^d is always negative. On the other hand, the sign of the second part relative to the fixed costs is mostly negative only when the initial value of the inframarginal rent is low. Recall that this part is composed of a negative value due to the risk-free rate effect and an ambiguous part due to the change in the cumulative distribution function. We find that the first negative part is almost not affected by a change of n^d, while on the contrary, the second part is always positive, but it significantly decreases with n^d with a lower value for a higher π_0. Indeed, the second part is linked to the probability of the fixed cost above the inframarginal rent. Hence a higher value of the rent always means a lower probability. All in all, the lower the probability, the higher the negative effect of the risk-free rate and hence the potentiality of n^d for having a negative effect on the capacity bid. From a policy perspective, it seems less costly to set up a short waiting phase for a profitable existing power plant and potentially allow a longer waiting time for less profitable investment.

An increase in the volatility of the inframarginal rent always increases the capacity bid in our numerical simulation. As illustrated in the second figure, higher volatility makes the capacity bids converge toward the periodic fixed cost of the investment, even though the initial value regarding the inframarginal rent is different. We also observe a diminishing marginal effect of the volatility on the capacity bid, meaning that it is sufficient for a slight increase from the current volatility to affect the bid significantly. Those results stem from the fact that the effect of the volatility on the inframarginal part of the bid is relatively small concerning the effect on periodic fixed cost. Indeed, the periodic fixed cost part directly includes the effect of σ on the total volatility of the revenue made during the transaction phase v, which is always positive. However, this effect is significant only for a low value of σ, which explains this diminishing margin effect. It has important implications, as, given this result, we should expect a rapid increase in the bids in capacity markets when the first effect of the introduction of renewable in the system will start to be significant. When renewables have a sufficient share in the production mix, the capacity price is assumed to be relatively stabilized.

7.3. Product design and capacity bid

We conclude our case study by studying the effect of segmenting a given capacity product into successive products with shorter transaction phases. We use an initial long product covering five years as the reference product, and we split this period into shorter periods. We use the same initial value of n^d of four years. We provide in figure 4 the results of the numerical simulation for the net present value and the real option. The point on the two figures represents four different product designs and k the number of successive products, with five yr. a single product covering the five years ($k = 1$), 1 yr. a yearly product ($k = 5$), semester a transaction phase covering six months ($k = 10$), and quarter a transaction phase covering three months ($k = 20$).

This simulation confirms the reverse effect of the choice of the product for the two frameworks: under the real options framework, the bids are in expectation lower with shorter products, while under the net present value framework, a more extended transaction phase always implies lower bids. For the latter case, this is explained by the possibility given by a more extended transaction phase to have positive revenue of a

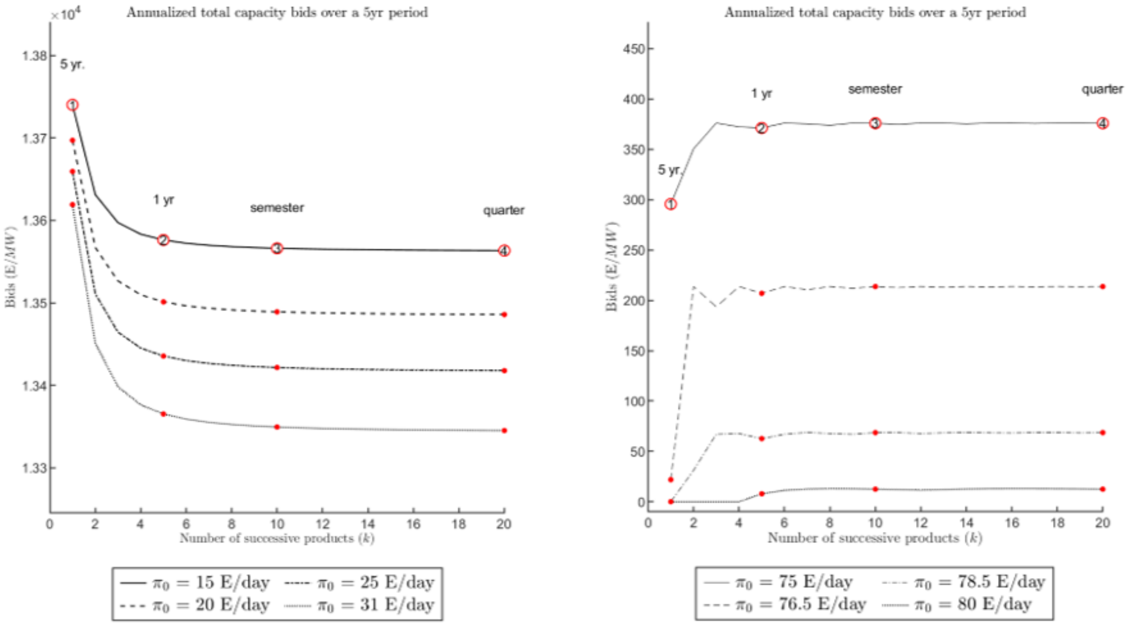


Figure 4: Evolution of the expected bids for a single long product and the sum of the expected bids for shorter periods.

specific period covering the potential loss incurred during another period in the transaction phase because of the fixed cost. This smoothing of the opportunity cost of participating in the capacity market is impossible for shorter products as they imply different opportunity costs and decisions. On the other hand, we do not find the same effect with the real options framework. Indeed, under this approach, the segmentation directly impacts the expectation of the option value for shorter periods, which is not the case under the net present value framework. To say it differently, the distribution characteristics of the total inframarginal rents over a long period are different from the sum of the distributions of the total inframarginal rents for successive shorter products. Given our numerical data, we find that segmenting the bids into shorter periods negatively impacts the inframarginal part and positively the fixed cost part of the bid. Given that the former is always negative and the latter is always positive, this segmentation continuously decreases the cost of a capacity market under the real options framework. Finally, our results also show that the product design choice is different from a marginal perspective by exhibiting a diminishing marginal effect for the two frameworks. We find that segmenting from a five-year period to a single-year period is sufficient to significantly decrease the bid under the real options framework or increase the bid under the net present value framework. Therefore, we show that it is unnecessary to make the capacity market over-complex under the real options framework by having many short products.

VIII. EXTENSIONS

We provide in this section a discussion on two relevant policy issues associated with the research questions of this paper. We first look at the implications of our analysis for bids in the capacity market for new power plants. Then we discuss the effect of the costs associated with the closing decision, which can only be assessed using our real options framework.

8.1. Product design and bids for new entrants

The relation between product design in this paper and bids for new entrants relies on the definition of the opportunity cost of entering the market as a new entrant. Indeed, as shown in equation 10, the bids should be equal to the NVP of the investment over the whole lifetime, including the two fixed costs (investment and operation) and the two sources of revenues (wholesale and capacity market). Therefore, assuming that the costs and the wholesale revenue are not impacted by the capacity market product design, any changes in the value of the bids when the investment is already in the market will impact the first bid even though we do not model the competition in the capacity market. It has clear policy implications. Indeed, if policymakers wish for more new entrants, it should aim at increasing the probability for those investments to be retained when they first bid into the capacity market[25]. Therefore, lower entry bids make this more likely to happen.

Following the previous analysis, we state in the lemma 3 this link between product design and bids for the new entry:

Lemma 3. *Under the net present value framework, shorter products imply that the initial bid B_0 for new investments is always lower or equal than the bid with a longer product.*

Under the real options framework, shorter products imply that the initial bid B_0 for new investments is always higher or equal than the bid with a longer product.

Proof. The proof is straightforward and is given by the results of proposition 1 and when the conditions of lemma 2 hold. □

To illustrate this discussion, we simulate the bid for a new entrant for a CCGT power plant using the previous data. We provide in figure 5 the relation between the initial value of the inframarginal rent and the capacity bid for a new entry and different capacity product designs. As expected, the figure shows that for shorter products, the bids in the capacity market is higher [26]. This figure also illustrates the sensitivity for a new power plant to enter the market when competing with existing investments. Indeed, we find that to provide a price even below the current price cap on the French capacity

[25] Note that a long waiting phase allows a new entrant to bid even though they did not build the power plant.

[26] As a higher inframarginal rent implies a lower bid, if for the same bid the threshold value π_0 is higher, it means that the capacity bids are higher for the same threshold.

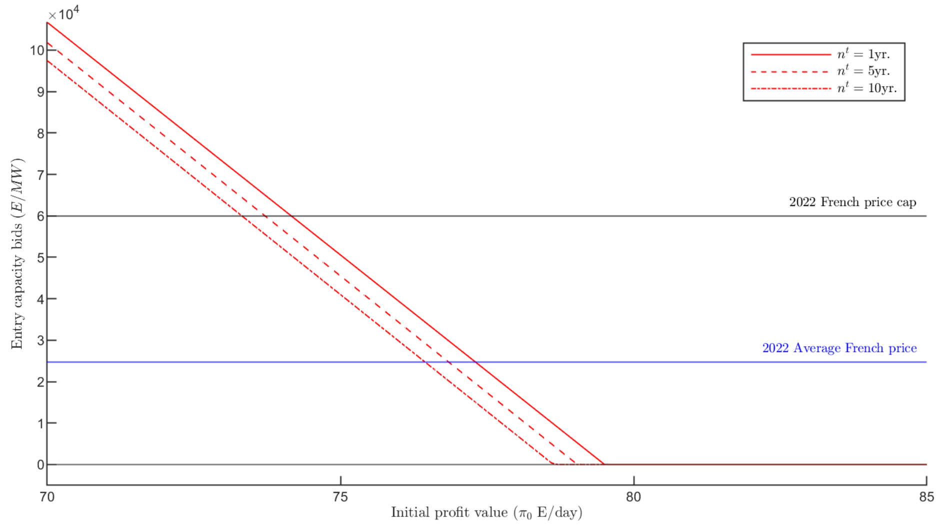


Figure 5: Capacity bid for new entrant with respect to the initial inframarginal rent

market, the producer needs to assume an initial value for π_0 of around 74 € / MWh for the three capacity products. On the other hand, as soon as the assumed inframarginal rent is above a value of 79 € / day, a new entrant always makes a null bid.

8.2. Penalty and mothballing costs

Finally, we discuss the effects of two drivers that increase the cost associated with the availability to close to avoid the periodic fixed cost. They both recover two distinct issues regarding the operation of an investment. However, they are identical in their conceptualization concerning the capacity bid analysis: (i) a policy instrument being the penalty associated with the failure to respect the obligation of being available by voluntary closing the investment (ii) the closing costs associated with the temporary shutdown of the power plant. They also have in common that a net present value framework cannot consider them in the capacity bid analysis. We summarise in lemma 4 their effects on the capacity bid under the real options framework.

Lemma 4. *Setting a penalty for the failure of not being available when a capacity product has been sold, or the existence of closing costs always leads to a lower bid in a capacity market. An increase in their value decreases the capacity bid.*

Proof. See Appendix □

The intuition behind those results is that the penalty or the closing cost decreases the value associated with closing to avoid the fixed cost. Therefore, they decrease the option value, hence the bid in the capacity market. Regarding the penalty value, it should be stressed that we consider in this section only the case when the power plant deliberately decides not to stay available, after observing a too low inframarginal rent, for instance. On the other hand, we left for future work when the penalty is applied because the power plant fails to stay available due to technical reasons. In this situa-

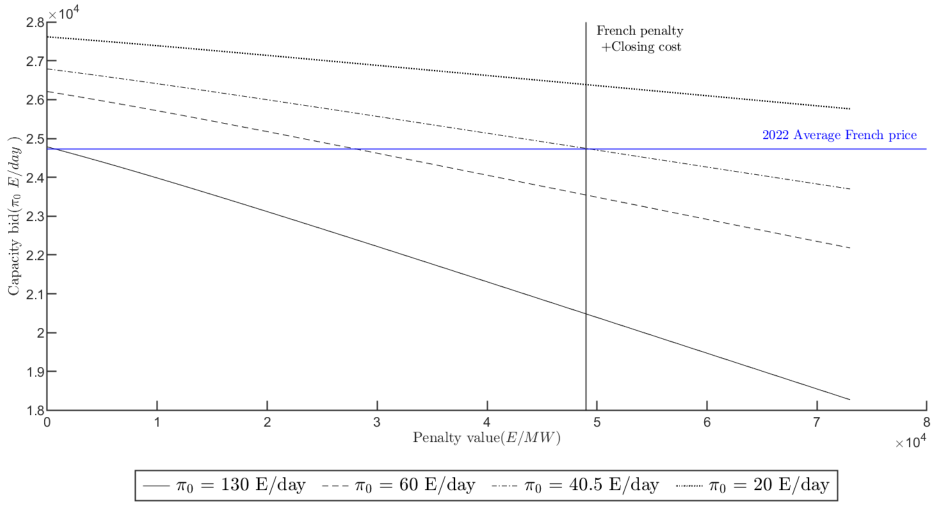


Figure 6: Evolution of the capacity price with respect to different penalty value

tion, a penalty increases the opportunity cost associated with participating in a capacity market, which increases the bid. [27]

We provide in figure 6 a numerical illustration of different values of the penalty on the bidding behavior in the capacity market. We assume that the closing costs are equal to 25% of the periodic fixed cost [1].

As expected, an increase in the penalty value decreases the bid in a capacity market. We show the current value in the French capacity market of 40000 € / MW in addition to the closing in the figure. To achieve the same price observed for the 2022 delivery year, we find that the initial value regarding the inframarginal rent needs to be equal to 40.5 € / day, which is almost half of the initial value to reach the same price without the penalty or the closing cost. Our numerical illustration shows the significant sensitivity of the choice of the penalty value when implementing the capacity market.

IX. CONCLUSION

In this paper, we provide a novel approach to analyze the bidding behavior in capacity markets. We distance ourselves from the framework of the net present value, which evaluates the bids on a capacity market as the net expected loss associated with the obligation to have the investment available on the wholesale market. While the net present value framework provides the fundamental rationales to understand the bidding behavior in capacity markets, it does not consider the value associated with the flexibility embedded in the investment. Using a real options framework allows us to consider both the uncertainty regarding the future revenue and costs of the investment and the intrinsic value associated with the alternative of participating in a capacity market, that is, to leave the market to avoid some fixed costs temporarily. We define the bid in a

[27] Such refinement of the model can be analyzed using the net present value framework and has already been studied on a Reliability Option mechanism by [26].

capacity market as the option value associated with this closing option, and we apply a pricing methodology of a Basket Option, an exotic derivative, to evaluate the real option value. Indeed, there is a similarity between this financial derivative, which allows receiving a basket of different asses prices against a strike price, compared to the managerial decision to stay open, receive revenues from the wholesale market periodically, and sustain an irrevocable fixed cost.

We use this framework to assess two issues related to the implementation and the design of capacity markets. First, we deepen our understanding of capacity markets and how prices are emerging on those competition-based mechanisms. Indeed, the use of a novel framework allows us to assess the determinant of the bids differently, as we have shown, for instance, for some drivers such as the waiting time between the auction and the beginning of the transaction phase or the volatility of the wholesale revenue. Therefore, any deviation from the actual value of the opportunity cost of participating in a capacity market can be better understood. It is particularly relevant in the current energy policy perspective as capacity markets are usually criticized for their additional burden onto consumers.

Then, we analyze the interplay between the product/commitment duration and the opportunity cost for providing capacity availability. We show that the choice of a product design can significantly affect the bids in a capacity market. Indeed, we find that a longer transaction necessarily implies a higher bid than shorter products under a real options framework. On the other hand, it is not always observed under a net present value framework. We also compare opposite market design regimes between having a long product sold in a unique auction or shorter products sold successively in the same period. We find that the outcome depends on various factors, but ultimately, it is more likely that the sum of bids for shorter transaction phases is lower than the individual bid for the long product. The opposite effect is observed for the net present value framework, making the real option one all the more relevant.

We deepen the policy implications of our work by simulating the bids in the French capacity market of a hypothetical investment in a CCGT power plant. It illustrates the theoretical framework while providing some additional analysis when a closed-form solution does not exist. We also discuss two straightforward extensions of our results. First, we show that the interplay between the choice of a market design regarding the capacity product has long-term effects by impacting the first bid in a capacity market for a new entrant. Second, we analyze the implications of having additional costs associated with the decision to close. We show that it can significantly modify the bids in a capacity market, which is not possible to assess using a net present value framework.

Our paper provides foundations for future work regarding the analysis of capacity markets. First, it would bring interesting empirical results to integrate this real options approach in a competition model. For instance, [27] has demonstrated that auction theory combined with real options can shed light on market outcomes, especially in the power sector. Second, capacity markets are closely related to the increase of new entries. A significant number of studies have tackled this issue, using real options theory, for instance, to assess the option value to enter the wholesale market [16]. However, to our knowledge, none have combined analysis of the entry decision with the participation in a capacity market with an endogenous bidding behavior under a real options framework. Finally, we have assumed a simplified representative investment with a single source of uncertainty: the inframarginal rent. On the other hand, it exists in current electricity markets various investments with different operational characteristics. For instance, renewables have high uncertainties regarding their output, while peak technologies face uncertainty in their merit order and production cost. New technologies, such as demand responses and batteries that are pushed to be integrated into capacity markets, also exhibit different uncertainty and operations. A refinement of the model by considering all of these characteristics would make it possible to underline future capacity markets implementations better.

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APPENDICES

A. PROOF OF PROPOSITION 2

We follow the demonstration from [27] and adapt it to the capacity market framework. The demonstration relies on the existence of a self-financing strategy Y_t between a bond process $D\beta_t = r\beta_t dt$ and the inframarginal rent process π_t, and on the assumption of an arbitrage-free market. Equating the coefficients of the self-financing strategy results in the following equation:

$$-rb^{opt}(\pi_t, t) + b_t^{opt}(\pi_t, t) + r\pi_t b_x^{opt}(\pi_t, t) + \frac{1}{2}\sigma^2 \pi_t^2 b_{xx}^{opt}(\pi_t, t) = 0 \tag{A.1}$$

It implies that we need to solve the following PDE:

$$-rb^{opt}(w, t) + b_t^{opt}(x, t) + rxb_x^{opt}(x, t) + \frac{1}{2}\sigma^2 x^2 b_{xx}^{opt}(x, t) = 0 \tag{A.2}$$

on the region $(x, t) \in (0, \inf) \times [0, T)$ with boundary condition $W(x, t) = \max(c^{om} - \pi_t, 0)$.

To solve this PDE, we introduce an equivalent risk neutral measure $\mathbb{Q}$ with $d\mathbb{Q} = Z_{\bar{T}} d\mathbb{P}$. Here $\mathbb{P}$ denotes the natural measure and $dZ_t = (\mu - r)\sigma^{-1} Z_t d\beta$. Girsanov theorem yields $d\beta_t = -(\mu - r)\sigma^{-1} dt + d\beta_t^{\mathbb{Q}}$ and therefore $d\pi_t = r\pi_t dt + \sigma\pi_t d\beta_t^{\mathbb{Q}}$. We can solve the PDE by applying the Feynman-Kac formula. For tractability, we assume that $t = 0$ and therefore $\bar{T} = n^d$. A solution is given by:

$$\begin{aligned} b_{opt}(\pi_t, 0) &= \mathbb{E}^*(-\int_0^{n^d} e^{rs} ds \max(c^{om} - \pi_t, 0)) \\ &= e^{rn^d}(c^{om} \int_{-\infty}^{c^{om}} dF(\pi_t) - \int_{-\infty}^{c^{om}} \pi_t dF(\pi_t)) \end{aligned} \tag{A.3}$$

The rest of the demonstration relies on computing the integrals. For that purpose, we note that $Y \sim \mathcal{N}(\omega, \zeta)$ and $X = e^Y$, then the distribution of X is:

$$F_x(x) = \phi(\frac{ln(x) - \omega}{\zeta}) \tag{A.4}$$

Where $\phi(.)$ is the cumulative density function of the standard normal distribution. Furthermore, the curtailed expected value of X is given by:

$$E[X|X < x] = e^{\omega+\frac{\zeta^2}{2}}\phi(\frac{ln(x) - \omega - \zeta^2}{\sigma}) \tag{A.5}$$

and

$$ln(\pi_{n^d}) \sim \mathcal{N}(ln(\pi_0) + (r - \frac{\sigma^2}{2})n^d, \sigma^2 n^d) \tag{A.6}$$

under the risk-neutral measure $\mathbb{Q}$

Using the identity $\phi(-x) = 1 - \phi(x)$, we have for the first integral:

$$\int_{-\infty}^{c^{om}} dF(\pi_{n^d}) = F_{\pi_{n^d}}(c^{om}) = \phi(-\frac{ln(\pi_0) - ln(c^{om}) + (r - \frac{\sigma^2}{2})n^d}{\sigma\sqrt{n^d}}) = \phi(z + \sigma\sqrt{n^d}) \tag{A.7}$$

And we define $z := -\frac{ln(\pi_0)-ln(c^{om})+(r+\frac{\sigma^2}{2})n^d}{\sigma\sqrt{n^d}}$. Which gives:

$$\int_{-\infty}^{c^{om}} dF(\pi_{n^d}) = \phi(z + \sigma\sqrt{n^d}) \tag{A.8}$$

For the second integral:

$$\int_{-\infty}^{c^{om}} dF(\pi_{n^d}) = \mathbb{E}(\pi_{n^d}|\pi_{n^d} < c^{om}) = \pi_0 e^{rn^d}\phi(-\frac{ln(\pi_0) - ln(c^{om}) + (r + \frac{\sigma^2}{2})n^d}{\sigma\sqrt{n^d}}) \tag{A.9}$$

Which gives:

$$\int_{-\infty}^{c^{om}} dF(\pi_{n^d}) = \pi_0 e^{rn^d}\phi(z) \tag{A.10}$$

With the expression of the integrals, we can express the option value associated with the possibility to close to avoid the fixed costs as:

$$b^{opt}(\pi_0) = -\pi_0\phi(z) + e^{-rn^d}(c^{om}\phi(z + \sigma\sqrt{n^d})) \tag{A.11}$$

B. PROOF OF PROPOSITION 3

We use the initial paper by [25] which approximates the distribution of the basket option by a log-normal distribution, and we apply it to the special framework of the capacity market.

Let $\pi(t)$ the inframarginal rent receives at time. We suppose that the sum of the inframarginal rent is determined on the interval $[\bar{T}, \bar{T} + n^t]$ which represents the transaction phase. We define the continuous sum as follow and we assume that $\bar{T}$ = 0:

$$\Pi_0 = \int_0^{n^t} \pi(t)dt \tag{B.1}$$

We look at characterizing the value of the *modified* basket put option. Using our capacity market framework, with notably the strike price equal to the sum of actualized periodic fixed cost C^{om}, it can be defined as:

$$P[\pi(t), \Pi_0] = e^{-rn^d}\mathbb{E}_0^*(max(C^{om} - \Pi_0)) \tag{B.2}$$

With E_0^* the expectation operator defined in the model section, which implies that under the risk-adjusted density function the inframarginal rent process can be described by $d\pi(t) = r\pi(t)dt + \sigma\pi_t dZ^*(t)$. For any value $t > 0$ we know that the value $ln(\pi(t))$ is normally distributed, with mean $ln(\pi_0) + (r - \frac{\sigma^2}{2})t$ and standard deviation $\sigma\sqrt{t}$.

The demonstration continues by assuming that the sum of log-normally distributed values Π_0 is indeed following a log-normal distribution, namely that $ln(\Pi_0)$ is normally distributed with a unknown mean m and variance v^2. Therefore, we use the moment generating function to determine those parameters. We define this function as $\Phi_x(k)$ with:

$$\Phi_x(k) = \mathbb{E}_0^*(\Pi_0^k) = e^{km + \frac{v^2}{2}k^2} \tag{B.3}$$

This expression allows us to consider a system of two equations with two unknowns, with the equations being the first two moments and the unknowns being m and v^2. Solving the system allows having the following expressions:

$$m = 2ln(\mathbb{E}_0^*[\Pi_0]) - \frac{1}{2}ln(\mathbb{E}_0^*[\Pi_0^2]) \tag{B.4}$$

$$v^2 = ln(\mathbb{E}_0^*[\Pi_0^2]) - 2ln(\mathbb{E}_0^*[\Pi_0]) \tag{B.5}$$

With $\mathbb{E}_0^*[\Pi_0]$ and $\mathbb{E}_0^*[\Pi_0^2]$ being the first and second moment of Π_0. Following our assumption regarding the process of the inframarginal rent, we can find a closed-form expression for the two moments . For the first moment, namely the mean of the sum, we can initially define it as follow:

$$\mathbb{E}_0^*[\Pi_0] = \int_0^{n^t} \pi(t)dt \tag{B.6}$$

Which gives:

$$\mathbb{E}_0^*[\Pi_0] = \pi_0 \int_0^{n^t} e^{rt} dt \tag{B.7}$$

For the second moment we use the the initial expression for two variables following a Geometric Brownian motion process , say $\pi(t1)$ and $\pi(t2)$. In this case, we have $\mathbb{E}_0^*[\pi(t_1)\pi(t_2)] = \pi_0^2 e^{r(t_1+t_2)+\sigma^2 t_1}$. Then we can expand the expression to a continuous framework and to the sum of the inframarginal rent, which gives:

$$\mathbb{E}_0^*[\Pi_0^2] = \pi_0^2 \int_0^{n^t} \int_0^{n^t} e^{r(t+s+n^d)+(s+n^d)\sigma^2} dtds \tag{B.8}$$

When assuming that Π_0 does follow a log-normal distribution, and with a closed-form expression for m and v^2, we can evaluate the put option $P[\pi_t, \Pi_0]$ using the standard finance theory as shown in the proof of proposition 2:

$$b^{opt}(\pi_0, \Pi_0) = P[\pi_0, \Pi_0] = -\pi_0 n^t \phi(z) + C^{om} \phi(z+v) \tag{B.9}$$

Where:

$$z = -\frac{m - ln(c^{om} \int_0^{n^t} e^{-rt}) + v^2}{v}$$

Note that we do not include any discounting factor for the inframarginal as it is already done using $n^t \pi_0$. Compared to the initial basket option, which compares asset price at the same period in time, in our framework, the option is exercised only with respect to the sum of the expected discounted inframarginal rent received during the transaction phase.

C. PROOF OF PROPOSITION 4

For the net present value, the derivative of the bid with respect to n^t is:

$$\frac{\partial b^{npv}}{\partial n^t} = -n^t + c^{om} e^{-r(n^t+n^d)} \tag{C.1}$$

Therefore the threshold for the sign of n^t on the netpresent value bid is given by the first order condition such that:

$$-n^t + c^{om} e^{-r(n^t+n^d)} = 0 \tag{C.2}$$

Which implies:

$$c^{om} = \pi_0 e^{r(nd+nt)} \tag{C.3}$$

For the real option bid, the derivative of the bid with respect to n^t is:

$$\frac{\partial b^{opt}}{\partial n^t} = -\pi_0(\phi(z) + n^t \frac{\partial z}{\partial n^t}\varphi(z)) + c^{om} e^{-rn^d}(e^{-rn^t}\phi(z+v) + \int_0^{n^t} e^{-rt} dt \varphi(z+v)) \tag{C.4}$$

When rearranged:

$$\frac{\partial b^{opt}}{\partial n^t} = -\pi_0(\phi(z) + n^t \frac{\partial z}{\partial n^t}\varphi(z)) + C^{om}(S_1\phi(z+v) + \varphi(z+v)\frac{\partial z + v}{\partial n^t}) \tag{C.5}$$

with $C^{om} = e^{-rn^d} c^{om} \int_0^{n^t} e^{-rt} dt$ and $S_1 = e^{-rn^t} / \int_0^{n^t} e^{-rt} dt$.

The Cdf ratio and the Df ratio conditions are given by rearranging again the equation and by respectively the first and second term in brackets:

$$\frac{\partial b^{opt}}{\partial n^t} = [C^{om} S_1 \phi(z+v) - \pi_0(\phi(z)] + \left[\frac{\partial z}{\partial n^t}(C^{om}\varphi(z+v) - n^t \pi_0 \varphi(z))\right] + C^{om}\varphi(z+v)\frac{\partial v}{\partial n^t} \tag{C.6}$$

Excluding the sign of $\frac{\partial z}{\partial n^t}$ the first two terms are positive if and only if: Cdf ratio: $\frac{S_1 C^{om}}{\pi_0} \geq R_0 = \frac{\phi(z)}{\phi(z+v)}$ Df ratio: $\frac{C^{om}}{n^t \pi_0} \geq R_1 = \frac{\varphi(z)}{\varphi(z+v)}$

The derivative $\frac{\partial v}{\partial n^t}$ is always positive and is equal to:

$$\frac{r\, \mathrm{e}^{\mathrm{nt}\,(r+\sigma)} + \sigma\, \mathrm{e}^{\mathrm{nt}\,(r+\sigma)} - r\, \mathrm{e}^{\mathrm{nt}\,r} - \sigma\, \mathrm{e}^{\mathrm{nt}\,(2\,r+\sigma)}}{2\,\sqrt{v^2}\, \left(\mathrm{e}^{\mathrm{nt}\,(r+\sigma)} + \mathrm{e}^{\mathrm{nt}\,r} - \mathrm{e}^{\mathrm{nt}\,(2\,r+\sigma)} - 1\right)} \tag{C.7}$$

Therefore, we need a third condition given by the sign of $\frac{\partial z}{\partial n^t}$. It can be express as follow:

$$\frac{\partial z}{\partial n^t} = -\frac{\frac{\partial v^2}{\partial n^t} + 2\,r}{2\,\sqrt{v^2}} - \frac{\frac{\partial v^2}{\partial n^t}\left(\ln\left(-\frac{\mathrm{com}\,\left(\mathrm{e}^{-\mathrm{nt}\,r} - 1\right)}{r}\right) - \frac{\ln\left(V^2\,p_0\right)}{2}\right)}{2\,v^3} \tag{C.8}$$

The sign of the derivative is given when equating the equation to 0, which given the following condition on the fixed cost for the derivative to be positive:

$$c^{com} \int_0^{n^t} e^{-rt} dt \geq \sqrt{\pi_0} V e^{v^2 (2r \frac{\partial v^2}{\partial n^t} - 1)} \tag{C.9}$$

The limits of the derivative at its extreme is found by analyzing the behavior of $\phi(z)$, $\phi(z+v)$, $\varphi(z)$ and $\varphi(z+v)$. Note first that $z \to 0$ when $n^t \to +\infty$, while $v \to +\infty$ when $n^t \to +\infty$.

Then the density function both converge towards 0 when $z \to 0$, $z+v \to +\infty$. For the cumulative density function: $\phi(z+v) \to 1$ when $z+v \to +\infty$, while $\phi(z) \to 0$. This implies the first result.

Concerning the case of of $n^t \to 0$, depending on the initial value of π_0 with respect to c^{om}, the value of $\phi(z)$ can either converge to 0 or to 1 when $n^t \to 0$. Indeed, recall that the sign of the derivative of z can either be positive or negative. However, in both cases, the value is either 0 or a positive value.

D. PROOF OF PROPOSITION 5

The results of the proposition follow directly from the derivative of the bid with respect to the variables.

For π_0:

$$\frac{\partial b^{opt}}{\partial \pi_0} = -n^t(\phi(z) + \pi_0 \frac{\partial z}{\partial \pi_0} \varphi(z)) + C^{om} \varphi(z+v) \frac{\partial z + v}{\partial \pi_0} \tag{D.1}$$

Which gives when rearranged:

$$\frac{\partial b^{opt}}{\partial \pi_0} = -n^t \phi(z) + \frac{\partial z}{\partial \pi_0}(C^{om} \varphi(z+v) - n^t \pi_0 \varphi(z)) + C^{om} \varphi(z+v) \frac{\partial v}{\partial \pi_0} \tag{D.2}$$

We found that: (i) $\frac{\partial v}{\partial \pi_0}$ is null and that (ii) $\frac{\partial z}{\partial \pi_0}$ is always negative as it is equal to:

$$\frac{\partial z}{\partial \pi_0} = -\frac{1}{p_0 \sqrt{v^2}} \tag{D.3}$$

Under the condition that $\frac{C^{om}}{n^t \pi_0} \geq R_1 = \frac{\varphi(z)}{\varphi(z+v)}$ (which is the Df ratio) then the derivative is always negative.

For c^{om}:

$$\frac{\partial b^{opt}}{\partial c^{om}} = -n^t \pi_0 \frac{\partial z}{\partial c^{om}} \varphi(z) + e^{-rn^d} \int_0^{n^t} e^{-rt} dt \phi(z+v) + \frac{\partial z + v}{\partial c^{om}} C^{om} \varphi(z+v) \tag{D.4}$$

Which give when rearranged:

$$\frac{\partial b^{opt}}{\partial c^{om}} = e^{-rn^d} \int_0^{n^t} e^{-rt} dt \phi(z+v) + \frac{\partial z}{\partial c^{om}} (C^{om} \varphi(z+v) - n^t \pi_0 \varphi(z)) + \frac{\partial v}{\partial c^{om}} C^{om} \varphi(z+v) \tag{D.5}$$

We found that: (i) $\frac{\partial v}{\partial c^{om}}$ is null and that (ii) $\frac{\partial z}{\partial c^{om}}$ is always positive as it is equal to:

$$\frac{\partial z}{\partial c^{om}} = \frac{1}{\mathrm{c^{om}} \sqrt{v^2}} \tag{D.6}$$

Under the condition that $\frac{C^{om}}{n^t \pi_0} \geq R_1 = \frac{\varphi(z)}{\varphi(z+v)}$ (which is the Df ratio) then the derivative is always positive.

For n^d:

$$\frac{\partial b^{opt}}{\partial n^d} = -\pi_0 n^t \frac{\partial z}{\partial n^d} \varphi(z) + C^{om}(-r\phi(z+v) + \frac{\partial z + v}{\partial n^d} \varphi(z+v)) \tag{D.7}$$

Which give when rearranged:

$$\frac{\partial b^{opt}}{\partial n^d} = C^{om}(\varphi(v+v) \frac{\partial v}{\partial n^d} - r\phi(z+v)) + \frac{\partial z}{\partial n^d} (C^{om} \varphi(z+v) - n^t \pi_0 \varphi(z)) \tag{D.8}$$

The conditions on the derivative of the bid with respect to n^d come straightforwardly. Note that the condition on the risk-free rate and the fixed costs are given by the respective derivative of v and z with respect to n^d:

$$\frac{\partial v}{\partial n^d} = -\frac{r - \sigma'}{2\sqrt{v^2}} \tag{D.9}$$

and

$$\frac{\partial z}{\partial n^d} = \frac{\left(\ln\left(-\frac{\mathrm{com}\left(\mathrm{e}^{-\mathrm{nt}\,r}-1\right)}{r}\right) - \frac{\ln(V)}{2}\right) \left(2\,r - \frac{\mathrm{e}^{-\mathrm{nd}\,\sigma}\,(r+\sigma)\,(V\,r+V\,\sigma)}{M\,p_0\,(\mathrm{e}^{\mathrm{nt}\,r+\mathrm{nt}\,\sigma}-1)}\right)}{2\,v^3} - \frac{\mathrm{e}^{-\mathrm{nd}\,\sigma}\,(r+\sigma)\,(V\,r+V\,\sigma)}{2\,M\,p_0\,(\mathrm{e}^{\mathrm{nt}\,r+\mathrm{nt}\,\sigma}-1)\sqrt{v^2}} \tag{D.10}$$

E. PROOF OF LEMMA 1

Recall that:

$$\frac{\partial b^{npv}}{\partial n^t} = -n^t + c^{om} e^{-r(n^t+n^d)} \tag{E.1}$$

and that:

$$\frac{\partial b^{opt}}{\partial n^t} = -\pi_0(\phi(z) + n^t \frac{\partial z}{\partial n^t}\varphi(z)) + C^{om}(S_1\phi(z+v) + \varphi(z+v)\frac{\partial z + v}{\partial n^t}) \tag{E.2}$$

Therefore the derivative of the flexibility, which is equal to the derivative of the difference between the two previous equation:

$$\frac{\partial \Gamma}{\partial n^t} = -\pi_0(\phi(z)+n^t \frac{\partial z}{\partial n^t}\varphi(z))+C^{om}(S_1\phi(z+v)+\varphi(z+v)\frac{\partial z + v}{\partial n^t})+n^t-c^{om} e^{-r(n^t+n^d)} \tag{E.3}$$

When rearranged:

$$\frac{\partial \Gamma}{\partial n^t} = -\pi_0((\phi(z)-1) + n^t \frac{\partial z}{\partial n^t}\varphi(z)) + C^{om}(S_1(\phi(z+v)-1) + \varphi(z+v)\frac{\partial z + v}{\partial n^t}) \tag{E.4}$$

The conditions and the ratio in the proposition stem directly from equating:

$$\frac{\partial \Gamma}{\partial n^t} = 0 \tag{E.5}$$

And from differentiating from the cases where the denominator of the ratio is positive or negative. Which is given by $S_1(\phi(z+v)-1) + \frac{\partial z+v}{\partial nt}\varphi(z+v) > 0$ or by $S_1(\phi(z+v)-1) + \frac{\partial z+v}{\partial nt}\varphi(z+v) < 0$.

F. PROOF OF LEMMA 4

We modify the results of the proof of proposition 2 and apply it directly to the proof of proposition 3. First, note that the new payoff of the basket option is now equal to $max(C^{om} - \Pi_0, -P)$, with P the costs associated with the closing decision, which are the penalty and the closing costs. In this case we need to solve the PDE of equation A.2

on the same region but with boundary condition $W(x,t) = \max(C^{om} - \Pi_t, P)$. We can solve the PDE by applying the Feynman-Kac formula. A solution is given by:

$$\begin{aligned} b_{opt}(\Pi_t, 0) &= \mathbb{E}^*(-\int_0^{n^d} e^{rs} ds \max(C^{om} - \Pi_0)) \\ &= e^{rn^d}(C^{om} \int_{-\infty}^{C^{om}+P} dF(\Pi_t) - \int_{-\infty}^{C^{om}+P} \Pi_t dF(\Pi_t) - P \int_{C^{om}+P}^{+\infty} dF(\Pi_t)) \end{aligned} \tag{F.1}$$

First note that for the two integrals, their expression are close to the one in the proof of proposition 2 and 3, namely we simply add the closing cost to the periodic fixed cost in the value z:

$$e^{rn^d}(C^{om} \int_{-\infty}^{C^{om}+P} dF(\Pi_t) - \int_{-\infty}^{C^{om}+P} \Pi_t dF(\Pi_t)) = -\pi_0 n^t \phi(z) + e^{-rn^d} C^{om} \phi(z+v) \tag{F.2}$$

with $z := -\frac{m - ln(c^{om} \int_0^{n^t} e^{-rt} dt + P) + v^2}{v}$.

For the third integral, recall that:

$$\int_{C^{om}+P}^{+\infty} dF(\Pi_t) = 1 - \int_{-\infty}^{C^{om}+P} dF(\Pi_t) = 1 - \phi(z+v) \tag{F.3}$$

Therefore:

$$b_{opt}(\Pi_t, 0) = -\pi_0 n^t \phi(z) + e^{-rn^d}(C^{om})\phi(z+v) - P(1 - \phi(z+v)) \tag{F.4}$$

When rearranged:

$$b_{opt}(\Pi_t, 0) = -\pi_0 n^t \phi(z) + e^{-rn^d}((C^{om} + P)\phi(z+v) - P) \tag{F.5}$$