The value of flexibility in power markets☆

Stéphane Goutte[a,c], Philippe Vassilopoulos[b]

[a] *Université Paris 8 (LED), 2 rue de la Liberté, 93526 Saint-Denis Cedex, France*
[b] *EPEX Spot SE, Product Design, 5 Boulevard Montmartre, 75002 Paris, France*
[c] *Paris School of Business (PSB) 59 Rue Nationale, 75013 Paris, France*

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ABSTRACT

The concept of flexibility is not one you find in standard microeconomics textbooks, yet it already plays a major role in the remuneration of the resources that generate and consume electricity every day and is likely to play an even larger role with the penetration of large intermittent renewable capacities. In this paper we attempt to quantify the net revenues that can be captured by a flexible resource able to react to the short term price variations on the day-ahead and intraday markets in Germany. We find that the difference between day-ahead and intraday revenues for a flexible resource has been increasing (although the profitability has been decreasing on both markets). This difference is more pronounced once 15 mn price variations can be captured by a flexible resource. The net revenues from the local 15 mn auction (which is held 3 h after the hourly "coupled" day-ahead auction) are more than eight times higher than the day-ahead hourly auction but below the net revenues that can be captured with the high prices from the continuous market. The results of the backward-looking empirical estimations allow us to distinguish and quantify two components of flexibility: (1) the "immediacy" value as we are approaching real-time and the urgency of the delivery increases (this value is revealed during the continuous intraday process and is highly linked to the stochastic nature of power supply and demand (i.e., wind/solar forecasts, forced outages of thermal generation,…) forecast error risk), and (2) the "ramping capability" component based on the technical characteristics as a resource can react to variations of shorter granularity (15 mn vs. 60 mn). We model and quantify the ramping capability component using a geometric brownian motion with jumps.

1. Introduction

As the share of intermittent resources is increasing in the power systems, balancing needs are exacerbated by large and variable generation ramps and forecast deviations of wind and solar adding to the traditional uncertainty of forced plant outages or demand. This is particularly true in the German electricity market where the installed capacity has reached unprecedented levels with more than 90 GW of installed wind and solar in 2015. Short term power prices should give the right signals to operate the existing resources and balance the power system (see Schweppe et al., 1988). The framework remains the same, although with a much larger uncertainty injected in the system on the supply side while the ability of consumers to react to price variations has not increased. Discrepancies between a variable supply and a very inelastic demand can lead to higher price volatility and price spikes on the short-term market.[1] Volatility of short term prices could provide additional revenue to the flexible resources able to react (i.e., on a 15 mn time step) and quickly (as real-time approaches).

In this paper we estimate the empirical ("backward-looking") and future ("forward-looking") net revenues that can be captured by a flexible resource, in our case a CCGT gas turbine, able to react to the short term price variations on the day-ahead and intraday markets in Germany and France. We find that the difference between day-ahead and intraday revenues for a flexible resource has been increasing (although the profitability has been decreasing on both markets).

This difference is more significantly higher once 15mn price variations can be captured by a flexible resource. Moreover, when we look at the net revenues that stem from the 15 mn auction that takes place every day ahead of delivery at 3 p.m. (for all 96 quarters of tomorrow's delivery) we find that there is a significant "ramping capability" premium compared to the hourly day-ahead auction (that takes place three hours earlier). These "backward-looking" empirical results allow us to quantify two different components for the value of flexibility:

1. The "*immediacy*" value as we are approaching real-time and the urgency of the delivery increases. This value is revealed during the continuous intraday process and is highly linked to the forecast error risk.
2. The "*ramping capability*" as a resource can capture variations of shorter granularity. This is more related to the technical characteristics of the asset and its optimization and can be priced already at the day-ahead stage through auctions for 15/30mn products and potentially 5 mn at a later stage.

In the second part of this paper, we analyze this "flexibility" component. We model the different short-term prices (hourly and quarterly auction prices) as mean-reverting jump diffusion (MRJD) process to understand the dynamics of flexibility revenues when the volatility and the number of jumps increase. The model parameters are estimated based on hourly and quarterly price time series. Results of the Monte Carlo simulations show that these revenues start low (1.18 (DE Hourly)–2.14 (DE Quarter) EUR/MWh) but increase rapidly between 2 and 5 year maturities (7.80–14.45 EUR/MWh). At 5 year maturities, auction revenues are seven times higher then their current levels. The revenues from the quarterly market are significantly higher than the hourly revenues.

These results highlight the importance of the intraday trading and the need to have sub-hourly granularity in the market to reveal the value of flexibility. Over time, although the value of immediacy is likely to decrease with improved forecasting techniques the need to have enough flexibility to account for the large generation ramps will not.

2. The flexibility challenge

Variability and uncertainty have always been common characteristics of power systems and managed by grid operators reluctantly with reserves. Because of the limited ability to store electricity, the low demand-side elasticity and the permanent need to match demand and supply, wholesale electricity prices can see extreme price volatility with positive and negative price spikes that can reach thousands of EUR/MWh in both directions (see Nicolosi, 2010 for an analysis of negative prices occurrences on the german market). Electricity is traded in Europe usually up to four years ahead of time either bilaterally or exchange based and until the real-time.

The German electricity market can be characterized as a self-scheduling market (as opposed to a central dispatch). Most short-term transactions (hourly and block products[2]) are hosted at a day-ahead auction every day ahead of delivery at noon. Continuous bilateral and exchange based trading is pursued until gate closure, typically 30 min before real time. In 2014 an auction for 15 mn contracts was launched (every day-ahead of delivery at 3 p.m. for all 96 quarters[3]) to complement continuous intraday trading.[4]

In 2015, an auction for quarters has been implemented to allow market participants to handle the optimization of their asset/customer portfolio already from the day-ahead (i.e., solar generation ramps that can be anticipated). In Central Western Europe (CWE) subsequently continuous bilateral and exchange based trading of hourly and 15 mn products is pursued until gate closure, typically 30 mn before real-time.[5]

Traditionally the intraday market has been used to allow market operators to manage their different resources sources of supply and their balance volume risk as a result of:

1. *Forced outages of generation units.* A power producer that has committed to selling the output of a power plant might need to buy the energy in the intraday if one of its plants becomes unavailable owing to a forced outage.
2. *Forecast error of demand.* A temperature decrease or cloudiness increase might require additional generation resources to meet load in real-time. The IDM can allow a supplier to purchase this missing energy.

With growing intermittent capacity additions,[6] the intraday market is increasingly needed to balance the volume risk associated with the high generation ramps and the forecast errors of intermittent generation. As explained by Garnier and Madlener (2015), sales made based on forecasts almost always require the seller to balance. In the absence of resources available within their own portfolios, market participants can turn to the intraday market to avoid imbalance penalties. Forecast errors remain significant (i.e., 5%) between the day-ahead gate closure and the real-time operations (see Vassilopoulos and Salah, 2013 for an analysis of intraday price and volume dynamics). As demand remains largely price inelastic, within-day variations in generation – even if accurately forecasted – need to be compensated by (1) changing the output from other plants (2) varying interconnection flows, or (3) curtailment of intermittent generation in the most extreme cases.

Intermittent renewable generation (iRES) imposes significant trading needs on market players seeking to balance their portfolios as real-time approaches.[7] Weber (2010) analyzes the relationship between the physical short-term balancing needs and the traded volumes on the intraday market. WIth a surge in wind and solar installed capacity, the forecasts error can account for a large amount of energy. In 2015, more than 37.5 TWh[8] were traded on the German intraday market. The following figure shows the evolution of trading volumes between 2009 and 2015 (Fig. 1).

The intermittent nature of solar/wind generation means that if there is not enough flexible capacity to ramp up and down as the intermittent solar and wind output varies, there can be supply discontinuities and price spikes (both negative and positive prices) can appear.[9] On the

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[2] Block orders are used to link several hours on an all-or-none basis, which means that either the bid is matched on all hours or it is entirely rejected. Block orders allow modelling the behavior of the power plant in the auction.

[3] The opening auction of the intraday market is similar to the auction at 12 a.m. For each 15 mn interval a single (uniform) clearing price is calculated and applied to all transactions for this interval.

[4] Most power markets in Europe still have hourly imbalance settlement step and therefore hourly market granularity. Markets with sub-hourly granularity area for example France and GB with 30 mn contracts and Germany, Austria and Benelux with 15 mn granularity.

[5] The EU power market is based on self-dispatch as opposed to central dispatch in the US. Market participants are encouraged/incentivized to self-balance unforeseen deviations from their day-ahead schedules.

[6] In Germany, in 2015 there were more than 75 GW of installed solar PV or wind capacity.

[7] The move to bring increasing iRES capacity additions closer to wholesale power markets and away from feed-in tariffs impose on iRES generators the balancing risks associated with the intermittency of their generation (See Mauritzen (2015)).

[8] https://www.epexspot.com/en/press-media/press/details/press/EPEX_SPOT_reaches_in_2015_the_highest_spot_power_exchange_volume_ever.

[9] For example, such discontinuities were observed during the solar eclipse on
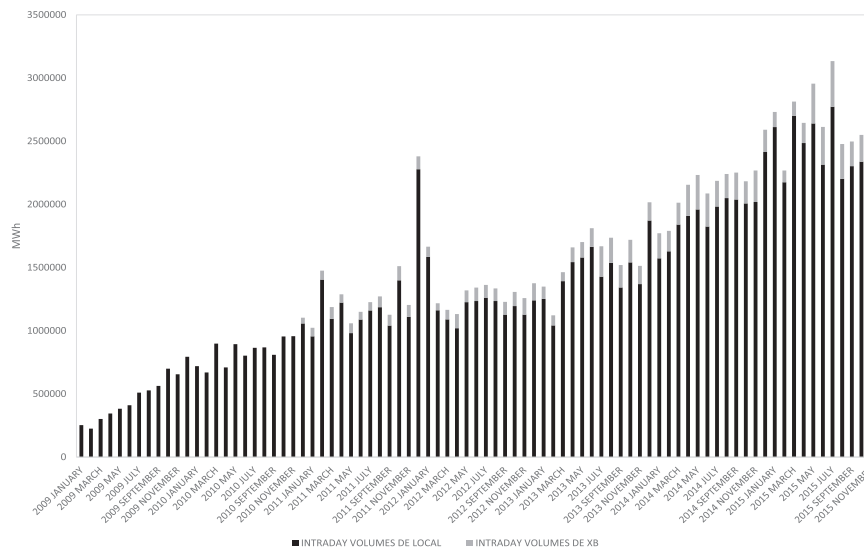
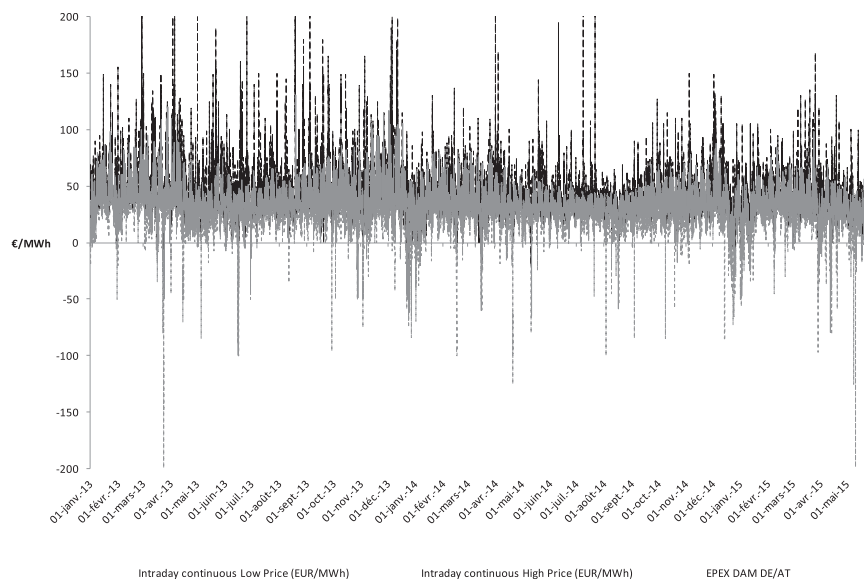**Fig. 1.** Evolution of intraday trading volumes in Germany between 2009 and 2015 (source: EPEX SPOT).

Fig. 2. Evolution of hourly wholesale prices on the day-ahead and Intraday market in Germany from 01/2013 to 06/2015 (source: EPEX SPOT).

intraday market, price limits are set at $[-9999 \text{ EUR/MWh}, +9999 \text{ EUR/MWh}]$ while on the day-ahead market prices cant exceed $[-500 \text{ EUR/MWh}, 3000 \text{ EUR/MWh}]$. Since 2007 the intraday prices for individual transactions have varied from a maximum of 5100 EUR/MWh to negative 1499 EUR/MWh for individual hours. Fig. 2 below shows the evolution of hourly wholesale prices on the Day-ahead and Intraday market (High/Low) in Germany from 01/2013 to 06/2015.

(*footnote continued*)

the 20th of March 2015 that created a significant drop in the solar output during the eclipse followed by a sudden increase back to its normal level as the eclipse arrived to an end. Instead of the usual morning/evening ramps, two additional ramps of solar generation appeared and had to be compensated by an increase in thermal generation quickly followed by a decrease in thermal generation. During the first ramp due to the solar eclipse the price of the given quarter jumped to 400 EUR/MWh. During the second ramp the price reached −200 EUR/MWh. The reader can find more information on this event: "European Power Exchange as a Component of Security of Supply during the Solar Eclipse" (Epexspot, 2015).

3. What is flexibility and why do we need it?

The power system needs flexibility in order to cope with the uncertainty associated with large-scale intermittent resource penetration.

1. At the system level, flexibility is the ability of the system to accommodate increasing levels of uncertainty while maintaining satisfactory levels of performance. System flexibility can be achieved if there are enough flexible resources in the system.
2. At the resource level, flexibility is the ability of to start-up quickly and adjust load output to changing market conditions.

There are several types of resources that can provide flexibility to the system. Some of them can provide more energy (upwards generation/incremental), Hydro and thermal generation plants, some can offer less energy such as wind turbines or demand side resources and some can offer both such as storage resources. Today demand-response capacity is still very limited in Europe and storage costs are still prohibitive for a large scale deployment. Garnier and Madlener (2016) explore the benefits that wind and photovoltaic power plant operators can

extract from the activation of flexible loads during their market operation on both day-ahead and intraday stages. We focus primarily on the profitability of existing gas power plants. The two most common flexibility parameters are:

1. *Start-up/shut-down time:* the time required for starting a plant to grid synchronization and actual contribution varies strongly from one technology to the other but in general it has significantly decreased over time. Historically for instance, large boilers required from several hours to couple of days to reach full output. This is suitable for baseload operation but under more volatile and uncertain conditions, new plants may be required to turn down and restart more frequently. If CTs and CCGTs have traditionally been faster to start than steam coal plants, several improvements are noticeable for all technologies. Most recent CCGTs have cut starting times by two and can reach full output in hot conditions in around 30 min. Older steam plants would require around 800 (or 14 h) minutes after an outage to reach full power, whilst most recent one can reach full output in 4 h after the same stop time. Some hydro plants can ramp-up and down in a matter of seconds. This is also true for some storage and Demand Side Management (DSM). (FTN): the modification of consumer demand for energy through various methods such as financial incentives.

2. *Minimum-up and down time*: If a thermal production is started up, for a number of technical reasons related to the operation of the thermal plant it should be on for a minimum number of hours known as the minimum up-time. If the unit is shut down it should remain down for a certain number of hours, known as the minimum down-time.

The addition of significant iRES generation is adding pressure on the profitability of conventional power plants by displacing generation with higher variable costs at the far right of the merit order curve. With the current fuel and carbon dioxide (CO_2) prices, the power plants that are most displaced in Europe are gas turbines. Several have announced early retirement or mothballing. As a result a number of gas plant operators are facing an unsustainable situation with plants running at a loss or at best hardly recouping fixed cost.

The iRES generation that comes online,[10] puts pressure on conventional power plant profitability by displacing generation with higher variable costs at the far right of the merit order curve or by contributing to the appearance of negative prices. With the current fuel and carbon dioxide prices, Europe's most displaced power plants are gas turbines (Combined Cycle Gas Turbines (CCGT) or Combustion Turbines (CT)). The plant revenues are below the fixed O&M level several have announced early retirement or mothballing. As a result a number of gas plant operators face an unsustainable situation with plants running at a loss or at best hardly recouping fixed costs. Although average prices have been falling it would be interesting to understand how volatility in short term prices could create the economic conditions to maintain enough flexible capacity to balance the power system.

4. An example of the need for flexibility

We present an example of a situation that required flexibility: the solar eclipse in Germany.[11] Solar generation for Friday March 20, 2015 was forecasted at 15 GW before the eclipse, falling below 7 GW at 10:30 a.m. and rising sharply above 22 GW around noon. Fig. 5 illustrates the volumes and prices for delivery Friday March 20, 2015 against a normal business day in March 2015. Normal business days see

important generation ramps within the day but the needs during March 2015 solar eclipse were unprecedented. The Day-Ahead auction and the 15 min auction are based on forecasts of the next day while the continuous markets allows market participants to balance their portfolios until 30 mn to real time. As illustrated in Figs. 3 and 4, power for delivery Hour 11 jumped in the Day-Ahead auctions to EUR/MWh 49.41 in Germany. On the 15 mn auction, during 11:00–11:15 a.m. a price of EUR/MWh 464.37 was observed and between 11:45 and 12:00 EUR/MWh −164.48. On the 15 min continuous intraday, there were also important price swings on the first and fourth contracts of Hour 11. The 11:00–11:15 transactions occurred at prices ranging from 35 to EUR/MWh 382.1, averaging at EUR/MWh 146.5. Most of the transactions of the 11:45–12:00 contract occurred at negative prices, with an average of EUR/MWh −2.7.

This extreme example illustrates well the interaction between the market and the supply demand balance on the grid with growing flexibility needs induced by the integration of large amounts of intermittent renewable energy

5. The value of flexibility

To estimate what revenues a plant could achieve, we have calculated the revenues for a combined-cycle gas turbine (CCGT) offered only on the DAM (case 1) or on the IDM. In order to empirically assess the revenues that a flexible resource can receive, we have computed the net revenues for a perfect MW from a Combined Cycle Gas Turbine (CCGT) able to ramp up and down based on day-ahead or intraday prices. If the price is above the variable cost of the CCGT the plant receives the difference between the market clearing price and its variable costs. If the price is below the variable costs the unit does not produce and has a profit of zero. This is a very simplified unit-commitment as we do not take into account the technical constraints of the power plant (min-up/down time, ramps, efficiency variations,…) nor the impact of the resource on the market clearing price. We have not considered the flexibility value of a demand-response resource or storage but this value can be computed in a similar way using technical parameters, cost estimates and wholesale power prices (price differentials in the case of storage). The same calculation is done for all markets, continuous or auction, hourly or quarterly.

– If the $A_t > VC_t$, the margin contribution is equal to $A_t − VC_t$.
– If the $A_t < VC_t$, the margin contribution is equal to 0.

The following Fig. 6 shows results when this calculation is done based on the DAM clearing prices and with the hourly intraday prices.[12] In the first case the CCGT is offered on the DAM only and in the second case the CCGT is offered on the IDM only (Fig. 7).

Although revenues are low in both markets due to the depressed power prices, the share of revenues from hourly Intraday compared to day-ahead has increased significantly[13] over recent years. In 2007, a flexible MW could earn 10% more net revenues by offering its MW to the Intraday market instead of a scheduling in the day-ahead. In 2015, a flexible MW can earn 70–80% more revenues by offering its MW on the intraday hourly market instead of the day-ahead hourly market.

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[10] Most renewable generation resources in Europe have been financed through Feed-in-tariffs. The amount of installed renewable capacity is independent of power prices and operators do not require price-spikes to recover their fixed costs.

[11] This example is derived from Epexspot ??

[12] For the day-ahead auction we use the market clearing price (hourly or 15 mn). As the intraday market is a continuous market we have used different proxies as the reference price. We use the Weighted Average Price (WAP), the High (highest price of the trading session) and the Low (lowest price of the trading session). For example, when we use the High make the assumption that the plant manages to get the highest hourly price during the trading session. High/Low overestimates/underestimate the actual revenues that could be earned by a CCGT on the intraday, but allow to determine benchmarks.

[13] There are several factors contributing to this decrease such as low coal/gas and CO_2 prices, depressed demand and the renewable capacity additions with low variable costs.
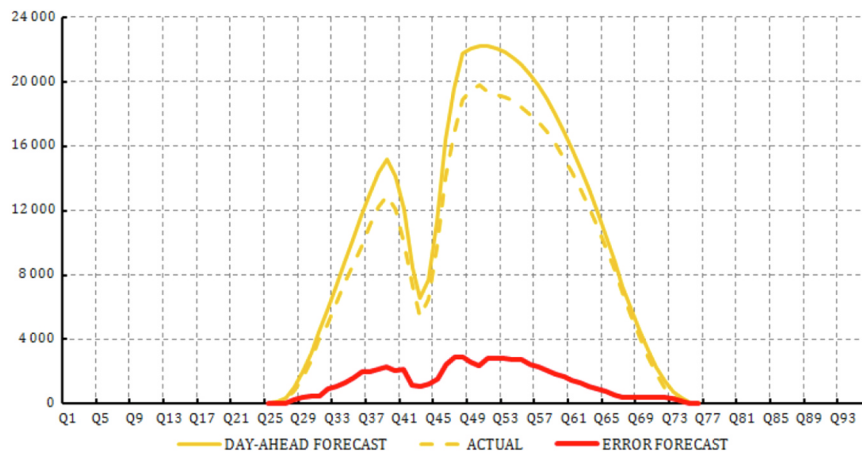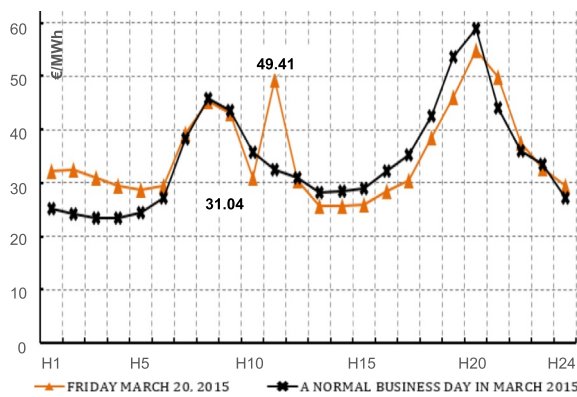
Fig. 3. Solar forecast.
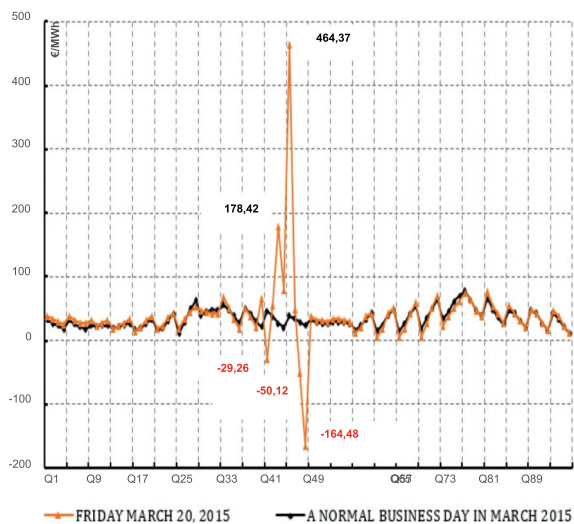

Fig. 4. Day-ahead prices.

Fig. 5. Intraday prices.

What happens if instead of hourly prices, the resource is scheduled based on 15 mn prices? A first interesting observation is that between 2013 and 2014 although the quarterly revenues have kept increasing hourly revenues have decreased. On/Off decisions of the plant at the quarterly level can be far more profitable as flexible resources benefit from a much higher price volatility at quarterly level. Although a perfect MW at the quarterly level is challenging to find in practice, it is interesting to see that it could increase revenues significantly on this quarterly price granularity. As shown in the table below which

summarizes results over a year, a perfect MW could increase its revenues by more than 60% compared to the hourly market from 59 k/MW-Yr to 95 k/MW-Yr.

In the auctions that take place on the day-ahead of delivery, the time lag is longer between gate closure and actual delivery and therefore the informational efficiency in these markets tends to be lower (Bellenbaum et al., 2014). However, it is interesting to look at the results of the same analysis with the 15 mn auction that takes place everyday in Germany at 3 p.m. (3 h after the day-ahead hourly auction) on the day-ahead for the 96 quarters of tomorrow's delivery. Instead of taking the price of the continuous 15 mn market we take the results of the 15 mn Intraday Call-auction. In Table 1, we can see that from the start of 2015 the net revenues received by the perfect CCGT MW on the 15 mn call-auction are 26,5 kEUR/MW-yr, always above the hourly revenues and between the high and WAP prices of the 15 mn continuous market. The difference with the net revenues that can be earned on the 15 mn continuous market remains very significant (almost 30%). The most rewarding market for a flexible resource remains getting the high prices on the 15 mn continuous market (33 kEUR/MW-yr in 2015), followed by the 15 mn call auction, the hourly continuous market and last but not least the Day-ahead hourly auction.

If we take into account technical constraints that would limit the flexibility of our perfect MW we see that the flexibility revenues decrease back to the level of the hourly market. If for example our perfect MW has minimum up-time constraints and needs to generate over consecutive several quarters before ramping-down, the revenues decrease towards the level of hourly Intraday revenues.

The net revenues of the plant under different minimum up-time assumptions (2 h, 3 h and 8 h). The outcome of these estimations is quite intuitive. An inflexible resource would lose the benefits in trading quarterly contracts.

These empirical estimations reveal two different components for what can be seen as "flexibility". The value of immediacy that can be estimated when comparing the net revenues that can be captured by a flexible resource on the continuous hourly market compared to the hourly day-ahead auction. In 2015, this value is 8162–4367 = 3795 EUR/MW-yr for the WAP and 33682–4367 = 29315 EUR/MW-yr for the High prices.

The net revenues obtained on the 15mn auction are lower than the revenues a market participant could get by offering a flexible resource on the continuous Intraday market and starting it closer to real-time but they are still eight times higher than the hourly auction (26445–4367 = 22078 EUR/MW-yr). This difference can be labelled "flexibility revenue" per se. This difference is also independent from the "immediacy" risk or forecast error risk that can be found on the continuous market. Even if the forecast was perfect, there might still be a need to ramp-up/down large capacity of back-up generation at 15 mn
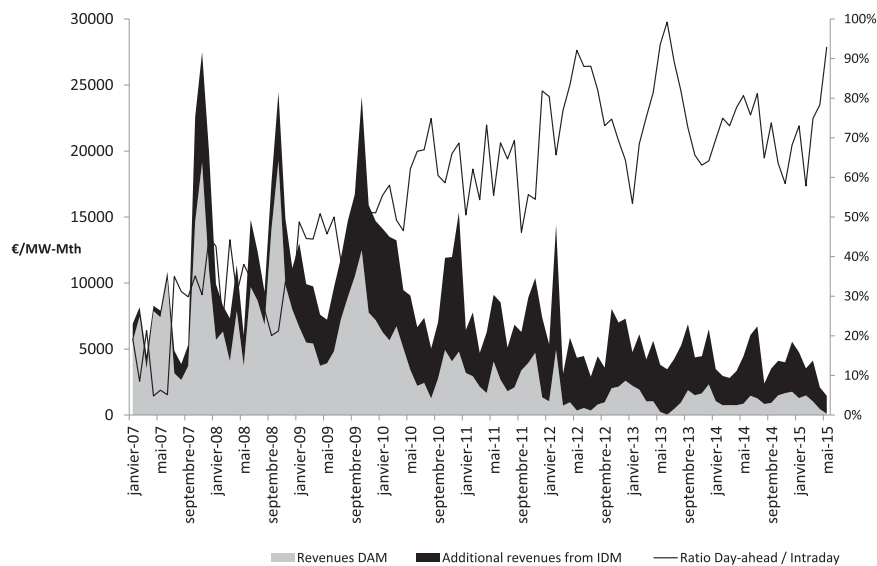
Fig. 6. DAM/IDM theoretical net revenues for a 1 MW perfect CCGT.

€/MW-yr	2013	2014	2015*
Intraday hourly (High)	59 729,39	49 436,61	17 833,67
Intraday hourly price (WAP)	19 629,12	16 812,89	4 732,02
Intraday hourly price (Low)	4 488,55	3 921,81	1 246,99
Intraday quarterly (High)	95 501,44	98 187,12	33 682,18
Intraday quarterly (Min up time 2hrs)	89 553,26	94 328,33	32 517,37
Intraday quarterly (Min up time 3hrs)	70 757,88	77 506,94	29 318,18
Intraday quarterly (Min up time 8hrs)	54 242,89	64 046,44	24 295,34
Intraday quarterly (WAP)	33 407,95	30 737,11	8 162,30
Intraday quarterly (Low)	6 191,27	5 203,61	1 212,21
Day-ahead market	15 282,19	13 590,19	4 367,46

Fig. 7. Recap of empirical simulation results for the simplified unit commitment of a 1 MW perfect resource (Source: Own calculations, EPEX SPOT, ICE).

Table 1
Theoretical net revenues for a perfect CCGT MW dispatched at the spot price.

€ /MW-yr	2013	2014	2015[a]
Day-ahead hourly auction	15282,19	13590,19	4367,46
Intraday continuous hourly price (high)	59729,39	49436,61	17833,67
Intraday continuous hourly price (WAP)	19629,12	16812,89	4732,02
Intraday continuous hourly price (low)	4488,55	3921,81	1246,99
Intraday 15 mn call-auction	–	–	26445,23
Intraday continuous 15 mn (high)	95501,44	98187,12	33682,18
Intraday continuous 15 mn (WAP)	33407,95	30737,11	8162,30
Intraday continuous 15 mn (low)	6191,27	5203,61	1212,21
Intraday 15 mn (min up-time 2 h)	89553,26	94328,33	32517,37
Intraday 15 mn (min up-time 3 h)	70757,88	77506,94	29318,18
Intraday 15 mn (min up-time 8 h)	54242,89	64046,44	24295,34

[a] Until June 2015.

steps. This is the value given, already at the day-ahead stage, by the ability to react to 15 mn price variations and can be priced in an auction. In the next section we investigate the statistical properties and model this "flexibility" value.

These "backward-looking" empirical results allow us to quantify two values for flexibility: (1) The "immediacy" value as we are

approaching real-time as the urgency of the delivery increases. This value is revealed during the intraday process and is highly linked to risk (2) the "flexibility" as a resource can capture variations of shorter granularity is more related to asset optimization and can be priced already from the day-ahead.

6. Modelling intraday price volatility and flexibility revenues

6.1. The stochastic model

In the next section we model the prices of electricity and try to quantify "flexibility".

6.1.1. Auction electricity model and variable cost

Let set a probability space $(\Omega, \mathcal{G}, \mathbb{P})$ where $W^{1,2} = (W_t^{1,2})_{t \in [0,T]}$ are two Brownian motion over a finite horizon $T < \infty$. Mean-reverting jump diffusion (MRJD) processes have provided the basic building block for electricity spot price dynamics since the very first modelling attempts in the 1990s. Their popularity comes from the fact that they address the basic characteristics of electricity prices (mean reversion and spikes), and at the same time are tractable enough to allow for computing analytical pricing formulas for electricity derivatives. Mean-

reverting jump diffusion (MRJD) models have also been used for forecasting hourly electricity spot prices and volatility. A mean-reverting jump diffusion model is defined by a continuous-time stochastic differential equation that governs the dynamics of the spot price process:

$$dA_t = (\alpha - \beta A_t)dt + \sigma dW_t^1 + Jdq_t, \quad 0 \leq t \leq T. \quad (6.1)$$

The Brownian motion W^1 is responsible for small (proportional to σ) fluctuations around the long-term mean $\frac{\alpha}{\beta}$. Thus W^1 represents the stochastic evolution of the future financial asset such as volatility, jumps… While an independent compound Poisson (jump) process q_t produces infrequent (with intensity λ) but large jumps of size J (here Gaussian with mean μ and variance γ^2). In this study it is reasonable to allow the intercept α to be a deterministic function of time to account for the seasonality prevailing in electricity spot prices.

Concerning the variable cost stochastic process VC, it appears that there is no jump component in its dynamic. Thus, this process evolves according to the following stochastic differential equation:

$$dVC_t = VC_t(\mu dt + \delta dW_t^2), \quad 0 \leq t \leq T. \quad (6.2)$$

6.2. Modelling prices before and after the default event

We are given a nonnegative and finite random variable τ, representing the default time, on $(\Omega, \mathcal{G}, \mathbb{P})$. Consider our two risky assets subject to possible default risk A_t and CV_t. If a default occurs before the maturity T then the volatility and/or jumps intensity components of the stochastic dynamics of the processes A_t and/or CV_t could be impacted. We can imagine that the volatility of the auction's price could increase of 20% dues to a crash of a nuclear central in our production.

Remark 6.1. The default time τ models default events which can occur during the time to maturity. This default time depends on several ecological, environmental and financial factors.

In our setting of possible default event, we have that the price processes A_t and CV_t are given by:

$$A_t = A_t^1 1_{t < \tau} + A_t^2(\tau)1_{t \geq \tau}, \quad 0 \leq t \leq T,$$
$$VC_t = VC_t^1 1_{t < \tau} + VC_t^2(\tau)1_{t \geq \tau}, \quad 0 \leq t \leq T.$$

where A^1 and VC^1 represent the dynamics of assets before the possible default event at time $\tau \in [0, T]$. These processes evolve, for all $0 \leq t \leq T$, according to following stochastic differential equations:

$$dA_t^1 = (\alpha^1 - \beta^1 A_t^1)dt + \sigma^1 dW_t^1 + J^1 dq_t^1, \quad A_0^1 = A_{0-}$$
$$dVC_t^1 = VC_t^1(\mu^1 dt + \delta^1 dW_t^2), \quad VC_0^1 = VC_{0-}.$$

where q^1 is an independent compound Poisson process with intensity of jumps λ^1 and large jumps of size J^1 (here Gaussian with mean μ^1 and variance $(\gamma^1)^2$).

Then $\{A_t^2(\tau), \theta \leq t \leq T, \tau \in [0, T]\}$ and $\{VC_t^2(\tau), \theta \leq t \leq T, \tau \in [0, T]\}$ are families of processes representing the dynamics of the assets after the default event occur at time $\tau \in [0, T]$ and governed for all $\tau < t \leq T$ by

$$dA_t^2(\tau) = (\alpha^2(\tau) - \beta^2(\tau)A_t^2(\tau))dt + \sigma^2(\tau)dW_t^1 + J^2(\tau)dq_t^2$$
$$A_\theta^2(\tau) = A_{\tau-}^2(1 + \gamma_\tau)$$
and
$$dVC_t^2(\tau) = VC_t^2(\tau)(\mu^2 dt + \delta^2 dW_t^2)$$
$$VC_\theta^2(\tau) = VC_{\tau-}^2(1 + \gamma_\tau)$$

Here, we have denotes by A_{0-} and CV_{0-} the initial values of assets and γ is a stochastic process valued in $[-1, \infty)$ and representing the jump of the asset A_{0-} and VC_{0-} at the default time τ.

Remark 6.2. The special case of a mothballing/retirement of an asset, the auction one A for example, after the default event τ is modeled with the case $\gamma_\tau = -1$ (and $\mu_t^2(\tau) = \sigma_t^2(\tau) \equiv 0$). Indeed, this means what

when the possible default time occurs the after default price A_t^2 will be equal at the time τ to

$$A_\theta^2(\tau) = A_{\tau-}^2(1 + \gamma_\tau) = A_{\theta-}^2(1 - 1) = 0$$

And so for all $\tau < t \leq T$

$$A_t^2(\tau) \equiv 0$$

The interpretation of the default risk model for an asset price is the following. The process A^1 (or VC^1) represents the asset price before the default, and there is a jump on the asset price at the default time, represented by the process γ, which may take positive or negative values, corresponding to proportional loss or gain on the asset price. After the default at time τ, $A^2(\tau)$ (or $VC^2(\tau)$) represents the asset price process, where there is a change of regimes in the coefficients depending on the default time. One typical situation can be as follows: in case of downward (resp. upward) jump in the asset price at default time $\tau \in [0, T]$, the rate of return $\mu^2(\tau)$ should be smaller (resp. greater) than the rate of return μ^1 before the default, and this gap should increase when the default occurs early, i.e. $\mu^2(\tau)$ is increasing (resp. decreasing) in τ with $\mu^2(\tau) <$ (resp. $>$) μ^1.

Remark 6.3. In our setting, there are several sources of randomness of both asset prices. Indeed, there are the financial market risk modeled with the Brownian motion W^1 and W^2; the Poisson jump component Jq_t of the Auction price process; there are the possible exogenous default event modeled with the stopping time τ, there are the stochastic jump at the default time τ modeled by $\gamma_\tau au$ and finally the regime switching shift of the parameters before and after the possible default event modeled by the shifting parameters.

6.3. Default event

In our framework, we assume that the default event τ is due to exogenous factors of the stochastic dynamics of the assets price A and VC. This means that the random variable τ is independent to the Brownian motion W^1, W^2 and the Poisson process q. Thus we get that there exists a deterministic function $f(\tau)$ of $\tau \in \mathbb{R}^+$ such that the survival probability is given by

$$G(t) = \mathbb{P}[\tau > t|\mathcal{G}_t] = \mathbb{P}[\tau > t] = \int_t^\infty f(\theta)d\theta \quad (6.3)$$

We assume, in the sequel, that the survival probability follows an exponential distribution with constant default intensity λ. So there is a constant $\lambda > 0$ such that $G(t) = e^{-\lambda t}$ and thus the density function is $f(\theta) = \lambda e^{-\lambda \theta}$. This probability implies that higher is the value of the default intensity λ, higher is the possibility of a default event in the dynamics of A and VC.

6.4. Payoff indicators

We decide to evaluate four financial indicators of our payoff at time $t \in [0, T]$,

$$\max(0, S_t - VC_t)$$

– At maturity: We calculate the classical payoffs value at maturity

$$\max(0, S_T - VC_T)$$

– Mean: We calculate the mean payoffs value between time $t = 1$ and maturity $t = T$. It corresponds to an average of profits.

$$\frac{1}{T} \sum_{t=1}^{t=T} max\left(0, S_t - VC_t\right)$$

– VaR: We evaluate the Value at Risk (VaR) of the maturity payoff. The VaR is an aggregated measure of the total risk of a portfolio of contracts and assets. The VaR summarizes the

expected maximum loss (worst loss) of a payoff contract over a target horizon (the maturity T) within a given confidence interval (generally 95%). Thus, VaR is measured in monetary units, Euros in our article. Maximizing the VaR is equivalent to increase our profit.

– CVaR: The Conditional Value-at-Risk (CVaR), is strongly linked to the previous risk measure (i.e., VaR) which is, as mentioned above, the most widely used risk measure in the practice of risk management. By definition, the VaR at level $\alpha \in (0, 1)$, $VaR(\alpha)$ of a given portfolio loss distribution is the lowest amount not exceeded by the loss with probability α (usually $\alpha \in [0.95, 1)$). The Conditional Value at Risk at level α $CVaR(\alpha)$ is the conditional expectation of the portfolio losses beyond the $VaR(\alpha)$ level. Compared to VaR, the CVaR is known to have better mathematical properties. It takes into account the possible heavy tails of portfolio loss distribution.

7. The data and numerical results

We use the Day-ahead and Intraday historical prices for Germany and France from 2012 to 2015.[14] For day-ahead we use 24 hourly prices per day. For the Intraday we use 96 quarterly prices from the 15 mn Call-Auction. The auctions allow a concentration of liquidity at a given point in time. The day-ahead auction which takes place at noon every day and the 15 mn call auction which takes place at 3 p.m. allow a robust reference price to emerge for all hours/quarters of the next day. In the German auction on average 700 GWh and on the quarterly auction 15–20 GWh are traded every day. Variable costs for a CCGT are calculated daily based on natural gas and CO_2 prices. They are based on the sum of fuel, emission and Variable O&M costs for a 1 MW CCGT. The payoff is the difference between the price and variable costs when price is above variable costs.

### 7.1. Parameters' estimation and descriptive statistics

7.1.1. Descriptive statistics

We estimate the historical volatilities of each market based on returns' prices.

$$\hat{\sigma} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (R_i - \bar{R})^2}$$

where R_i denotes the ith returns' prices, $R_i = A_i - A_{i-1}$ (or $R_i = VC_i - VC_{i-1}$ for Variable Costs), $\bar{R}$ is the empirical mean: $\bar{R} = \frac{1}{N} \sum_{i=1}^{N} R_i$ and N denotes the total number of observation.

We have a historical profit of 1.1310 euros per MWh for hourly auction german's market and 1.7258 euros per MWh for quarterly auction german's one. In the figure below, we plot the historical data payoff evaluate on the Hourly german's auction prices (
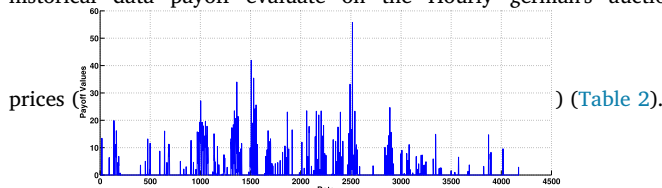
) (Table 2).

7.1.2. Parameters' estimation

We estimate the parameters of the three price series (Table 3).

α and β significantly higher for the quarterly auction, same for γ and $lambda$ the variance of the jumps and their intensity. The model volatility is higher for the hourly german power prices (Table 4).

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[14] EPEX SPOT Day-ahead auction and EPEX SPOT Intraday Continuous.

Table 2
Descriptive statistics of the data.

Markets				
	Hourly German	Quarterly German	Hourly France	Variable costs
Mean	29,8732	29,8170	32,1707	42,9825
Median	29,5200	29,5500	33,5137	42,8108
Variance	185,4167	274,0573	123,7806	3,9322
Skewness	−0,5177	0,7572	−0,0485	0,1522
Kurtosis	6,4304	34,2469	3,1646	2,2572

Table 3
Historical volatilities.

Markets				
	Hourly German	Quarterly German	Hourly France	Variable costs
$\hat{\sigma}$	5.2784	14.0843	4.1830	0.1322

Table 4
Parameters' estimation.

Market	α	β	σ	μ	γ	λ
Hourly Germany	1,2893	0,0637	13,9939	11,5661	3,0406	0,0394
Quarterly Germany	11,6492	0,3826	13,7145	367,0112	4,8053	0,0569
Hourly France	4,9449	0,1198	11,7719	41,2667	1,0000	0,0030

7.2. Numerical results

The Monte Carlo simulation[15] procedure applied takes random draws from the distributions of both the electricity price and the variable cost of the CCGT. We perform the simulations for both the Day-ahead hourly auction and the 15mn Call-Auction price distributions for different maturities. The Table 5 summarizing the results for the hourly and quarterly prices is shown below

For the three series the patterns appear quite similar. Results of the Monte Carlo simulations show that these revenues start low (1.18 (DE Hourly)–2.14 (DE Quarter) EUR/MWh) but increase rapidly between 2 and 5 year maturities (7.80–14.45 EUR/MWh). At 5 year maturities, auction revenues are eight times higher then their current levels.

When we compare the results for Germany with France, for the one month maturities, the average payoff ranges from 1,24 (FR hourly) to 1,18/MWh (DE hourly) and for the five year maturities from 9,68 to 7,80/MWh.

It is interesting to see that between two and five years maturity, the payoff more than doubles for the three series, For the quarterly auction, the payoff is almost multiplied by a factor of three. What happens if we increase volatility and jump parameters?

7.2.1. Impact of volatility and jump at default time

We model the short-term prices both hours and quarters as mean-reverting jump diffusion (MRJD) processes. The following tables summarize the numerical results for the German hourly and quarterly auctions when the volatility and jump parameters vary. As expected, for all cases the payoff of the flexible MW increases as the volatility and jump parameters increase at default time. However, they do not grow at the same rate. When volatility and jumps increase by 100%:

– the payoff of the hourly auction increases from 1,2/MWh (standard mean) to 5,1/MWh (mean) and to 6,9/MWh for CVaR.

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[15] We used 5000 simulations in our results since we tested a range of number of simulations between 1000 and 10,000 to confirm this choice regarding the convergence of the results.

Table 5
Day-ahead hourly auction and the 15 mn call-auction price distributions for different maturities.

		Maturities					
		One month	Two months	Six months	One year	Two years	Five years
Hourly	T	1.3426	1.4274	2.1530	3.6655	6.2740	13.9554
Germany	Mean	1.1824	1.2802	1.6579	2.3701	3.7330	7.7968
	VaR	1.3227	1.4949	2.1984	3.5536	6.2273	13.5885
	CVaR	1.3545	1.5264	2.2819	3.6389	6.3527	13.9075
Quarterly	T	2.2247	2.5715	3.6244	5.5253	8.8871	18.8654
Germany	Mean	2.1426	2.3801	2.9515	3.8024	5.5071	14.4534
	VaR	2.3019	2.6773	3.7040	5.3328	8.4945	18.4397
	CVaR	2.3365	2.7379	3.7842	5.4551	8.6848	18.9235
Hourly	T	1.4137	1.5898	2.6080	4.4691	7.9749	17.1619
France	Mean	1.2439	1.3822	1.8941	2.8458	4.6306	9.6757
	VaR	1.4088	1.6517	2.6129	4.4092	7.8386	16.6777
	CVaR	1.4385	1.6928	2.7060	4.5131	7.9829	17.0244

– the payoff of the quarterly auction increases from 2,14/MWh (standard mean) to 7,6/MWh (mean) and to 9,7/MWh for CVaR

The revenues from the quarterly market are significantly higher than the hourly revenues. Profitability increases as volatility and jump parameters increase.

As can be seen in Table A1, when increasing price volatility and intensity jumps unlike profits in terms of margin differences between 60 mn/15 mn auction decrease. This is because we end up with two markets that are both very volatile and very spiky. This mitigates their different dynamics. It was indeed a smoothing phenomenon specific properties to these two markets towards a single market.

7.2.2. Differences between hourly and quarterly revenues

We then compare the differences between the hourly and quarterly prices (see Table A3). If volatility and jumps increase from 0% to 100%, the difference decreases from 81% to 49%. Thus, increasing the jump process tends to smooth differences between quarterly and hourly auctions (where jumps are currently very modest). It is interesting to note that this difference tends to strongly decrease with jumps but increase with volatility, up to 50%. For example, when we increase volatility by 50% the difference between hourly and quarterly payoff grows from 81% to 105% but an increase of 100% brings the difference down to 42%.

When we increase the price volatility and the intensity of jumps, the difference in net revenues between 60mn/15 mn auctions increases with increases of volatility of 10% and 50% and decreases with the intensity of jumps. At first sight, this seems counter-intuitive. One explanation is that we end up with two markets that are both very volatile and very "spiky". The spikes/jumps are very powerful in increasing the net revenues; This mitigates their different dynamics between 15 and 60 mn auction prices. It is indeed a smoothing of the specific properties of each market towards a market with common/similar properties

8. Conclusion and policy recommendations

We have seen that a flexible plant can increase its revenues by ramping up/down close to real-time and/or quarter by quarter. Volatility of short term prices provides an additional revenue to the flexible resources able to react quickly as real-time approaches. We have found that the empirical "backward-looking" net revenues that can be captured by a flexible resource able to react to the short term price variations in the day-ahead and intraday markets. We find that although the profitability has been decreasing on both markets, the difference in profitability between day-ahead and intraday markets has been increasing significantly. This is even more pronounced when we look at 15 mn prices. Already from the day-ahead stage, in the 15mn auction, this flexibility (the ability to tamp up/down at 15mn intervals) has a strong value/premium compared the hourly auction.

These "backward-looking" empirical results allow us to quantify two values for flexibility:

1. The "immediacy" value as we are approaching real-time as the urgency of the delivery increases. This value is revealed during the intraday process and is highly linked to risk.
2. The "flexibility" as a resource can capture variations of shorter granularity is more related to asset optimization and can be priced already from the day-ahead.

In the second part of this paper, we quantify this "flexibility" component. We model the prices of the two auctions that take place in the day-ahead of delivery (hourly and 15mn) as mean-reverting jump diffusion (MRJD) process to understand the dynamics of flexibility revenues as volatility and number of jumps increase. Results of the Monte Carlo simulations show that these revenues start low (1.2–2.2 EUR/MWh) but increase rapidly between 2 and 5 year maturities (7.8–14.4 EUR/MWh). At 5 year maturities, auction revenues are seven times higher then their current levels. The revenues from the quarterly market are significantly higher than the hourly revenues. As expected, profitability increases as volatility and jump parameters increase, however it is interesting to observe that the difference between the hourly and quarterly net revenues tends to increase with volatility (up to a 50%) but decreases with jumps.

This analysis highlights the importance of the Intraday market to economically maintain enough flexible resources to back the intermittent generation. More granular products such as 15/30mn (and potentially 5mn at a later stage) should be generalized to improve the price signal for the operation of the power system but also to guide the investments in the countries that are likely to see a strong growth in intermittent generation capacity over the coming years. We can expect the value of immediacy to decrease with improved forecasting techniques but the need to have enough flexibility to account for the large generation ramps will increase.

Appendix A

(Table A2)

Table A1
Hourly payoff for variations of the volatility and jump parameters at default time.

		Jumps increase 0%	Jumps increase 10%	Jumps increase 50%	Jumps increase 100%
Standard Increase Vol 0%	T	1.3426	1.5909	2.2193	2.8774
	Mean	1.1824	1.3924	1.8542	2.3397
	VaR	1.3227	1.5906	2.2788	3.0004
	CVaR	1.3545	1.6179	2.3114	3.0307
Increase Vol 10%	T	1.5431	1.6958	2.5716	3.1813
	Mean	1.3477	1.5533	2.2381	2.7434
	VaR	1.5409	1.7423	2.5675	3.2488
	CVaR	1.5683	2.2332	2.5967	3.2921
Increase Vol 50%	T	2.2932	3.3453	4.2421	5.7704
	Mean	1.9917	2.7897	3.7735	5.1319
	VaR	2.3650	3.3666	4.2718	5.7364
	CVaR	2.3952	3.4117	4.3270	5.8085
Increase Vol 100%	T	5.0448	5.1444	5.9250	6.5956
	Mean	4.1317	4.6307	4.4664	5.0751
	VaR	5.0638	5.1648	5.9516	6.8170
	CVaR	5.1273	5.2351	6.0276	6.8823
Increase Vol −50%	T	0.4349	0.6368	1.0943	1.6767
	Mean	0.4548	0.5365	1.0610	1.4688
	VaR	1.0925	0.6209	1.1686	1.7689
	CVaR	1.1946	0.6508	1.1846	1.7930

*Until 15th of June.

Table A2
Quarterly payoff for variations of the volatility and jump parameters at default time.

		Jumps increase 0%	Jumps increase 10%	Jumps increase 50%	Jumps increase 100%
Standard Increase Vol 0%	T	2.2247	2.0390	2.6683	3.1884
	Mean	2.1426	2.0987	2.3992	3.1221
	VaR	2.3019	2.2830	2.8618	3.3100
	CVaR	2.3365	2.3399	2.9177	3.3521
Increase Vol 10%	T	2.6491	2.7522	3.6851	4.5307
	Mean	2.6560	2.5423	3.5763	4.3488
	VaR	2.8461	2.7567	3.8049	4.5662
	CVaR	2.8814	2.8025	3.8515	4.6094
Increase Vol 50%	T	4.8098	4.9516	5.6939	6.7833
	Mean	4.0551	4.4262	5.1494	6.7155
	VaR	4.9142	5.1243	6.0987	7.0976
	CVaR	5.4526	5.1907	6.4312	7.1572
Increase Vol 100%	T	6.8524	7.8182	8.3897	9.2768
	Mean	5.8975	5.6877	6.3145	7.5690
	VaR	7.3780	8.1734	8.6544	9.5712
	CVaR	7.4799	8.2612	8.7519	9.6534
Increase Vol −50%	T	0.7438	0.8445	1.3487	2.4310
	Mean	0.9623	0.8527	1.8126	1.9096
	VaR	2.2062	0.9317	2.0208	2.3866
	CVaR	2.2589	0.9672	2.0626	2.8343

Table A3

Differences of profits between hourly and quarterly revenues for variations of the volatility and jump parameters at default time.

		Jumps increase 0%	Jumps increase 10%	Jumps increase 50%	Jumps increase 100%
Standard Increase Vol 0%	T	0.6570	0.2817	0.2023	0.1081
	Mean	0.8120	0.5073	0.2939	0.3344
	VaR	0.7403	0.4354	0.2558	0.1032
	CVaR	0.7250	0.4462	0.2623	0.1060
Increase Vol 10%	T	0.7168	0.6230	0.4330	0.4242
	Mean	0.9707	0.6368	0.5979	0.5852
	VaR	0.8470	0.5822	0.4819	0.4055
	CVaR	0.8373	0.2549	0.4832	0.4001
Increase Vol 50%	T	1.0975	0.4802	0.3422	0.1755
	Mean	1.0360	0.5866	0.3646	0.3086
	VaR	1.0779	0.5221	0.4277	0.2373
	CVaR	1.2764	0.5214	0.4863	0.2322
Increase Vol 100%	T	0.3583	0.5197	0.4160	0.4065
	Mean	0.4274	0.2283	0.4138	0.4914
	VaR	0.4570	0.5825	0.4541	0.4040
	CVaR	0.4589	0.5780	0.4520	0.4026
Increase Vol −50%	T	0.7102	0.3261	0.2326	0.4498
	Mean	1.1158	0.5894	0.7084	0.3001
	VaR	1.0194	0.5007	0.7293	0.3493
	CVaR	0.8910	0.4861	0.7412	0.5808

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